SUPPORTING MATHEMATICAL PRESENTATIONS BY CAS-USE FROM CLASS 7

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The five-year model test CAliMERO[1] is evaluating the use of computer-algebra-systems from class 7 in connection with a teaching concept which concentrates on sustainable learning and the development of mathematical competencies. First research results show that especially low attainers are able to reach above-average performance increases with this concept.

DESCRIPTION OF THE PROJECT

On the basis of the experiences made with graphics calculators in the German Federal State of Lower Saxony the school project started in summer 2005 with the aim to introduce the reasonable use of CAS-calculators in secondary school level I. To reach this target it is planned to develop a curriculum and design concept for maths lessons where a new tasks culture is established and the calculator is used for the enhancement of mathematical competencies. The project CAliMERO was started in the school year 2005/2006 in six Gymnasiums with 29 classes of level 7 which are working very closely with the developed lesson elements. In the current school year they are already 50 schools to use the material developed and tested the year before. In the next years CAliMERO will be continued up to class level 10. In order to enhance sustainable maths learning with CAS it is necessary, as described by Stacey (2003), to establish a teaching culture which corresponds to the use of CAS. Therefore a further training course of several days took place at the beginning of the project with representatives of the participating schools, experts from Lower Saxony and under the direction of Prof. Dr. Regina Bruder. There were discussions about appropriate teaching methods to support the development of competencies in CAS-supported lessons according to the German education standards (KMK, 2003). The teaching concept developed with the participating teachers intends to make use of the complex potential of calculators for the discovery of maths and for effective exercises for a better understanding. Additional meetings during the project are organized every three months to improve communication between the participants, to develop the next teaching elements and learning materials for the students and to discuss the state of evaluation. Moreover the TU Darmstadt offers project coaching by means of a special internet platform which allows the ideas exchange of the participants and contains all developed materials(www.prolehre.de).

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[1] Computer-Algebra in maths lessons – discovering, calculating, organizing

EVALUATION CONCEPT OF THE PROJECT

The evaluation of the project CAliMERO is focussed, individually and in mutual correlation, on the implementation of the defined targets, specific disposition and teaching culture of the involved teachers and on the perception of the lessons by the students as well as on the development of the maths performance of students.

To evaluate the implementation of the project targets the material developed by the teachers (task-based teaching elements) is analysed with respect to its consistency and conformity with the teaching concept. The work products of students resulting from the lessons are investigated under the aspect to what extent the different display formats of mathematical correlations offered by CAS were adopted. In the first year the teachers filled in a standardized lesson report after every lesson with CAS-use. Additionally to its monitoring function this method offered incentives for the realization of the concept. In the second year of the project the lessons are documented by students of level 8 with a standardized form. The form records the content and design of the lesson, the extent and purpose of the CAS-use and the course of the lesson from the students' point of view. Moreover one student of the class describes a maths problem treated in the lesson with the discussed possibilities of solution. An annual questionnaire analyses the development of the teachers with respect to their perception of the lessons, the use of CAS and their own role as a teacher. They are also asked to document their ideas of lesson quality. After the introduction of every teaching element an evaluation form is presented to the teachers to comment the curricular implementation and occurring problems. This direct feedback allows the prompt revision and improvement of the teaching material for the classes to come.

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The performance development of the learners is analysed with an annual pre-post-test. This test is also run in comparison classes where traditional teaching concepts and graphics calculators are applied. The attitude of the learners towards maths and the use of calculators is recorded in an annual questionnaire for students, highlighting the perception of maths and maths lessons, the learning motivation and the acceptance of calculators.

PRESENTATION OF THE PERFORMANCE TESTS

Format of the student performance tests

The tests which were conceived as open end tests include more and less demanding tasks in alternating order. In class level 7 the test is run without calculator support, from class 8 the use of calculators is admitted and the CAS-potential of tasks is evaluated by comparison with classes working with graphics calculators.

In the school year 2005/2006 the pre-post-performance test for class level 7 was run with 26 items in two parallel groups. It examined mathematical guiding principles like numbers, measuring, space and form, functional correlation of data and

coincidence with special focus on the handling of numbers and functional correlations. At the beginning of school year 2006/2007 a performance test for class 8 was run in two groups again with three anchor tasks from level 7 which will be repeated as post-test with changed groups at the end of the school year.

Competency orientation of the test tasks

An analysis of the possible algorithms of the different tasks allowed to determine their potential with respect to the general mathematical competencies defined in the German education standards for maths which are examined in the performance test of students. Of special interest to the project CAliMERO is the development of the learners concerning their modelling competency, the use of mathematical arguments, their problem-solving abilities, the use of mathematical presentations and the handling of symbolic, formal and technical maths elements, as these can be enhanced by the use of CAS in maths lessons. Handling of mathematical presentations includes, for example, the establishing of relations between different presentation forms and their interchange. Insofar the project takes up the results by Amit and Fried (2005) who showed that ”we have to challenge a multiple representations approach as a framework to begin with in teaching“. Dealing with symbolic, formal and technical maths elements includes the use of mathematical terminology and the handling of formulas and symbols. The technical aspect of this competency is the selection and application of mathematical tools like the functioning of a CAS-instrument and the reflection on the possibilities and limits of their use.

The applied performance tests are assessing the competence-related development of the learners with problem-solving tasks requiring the mathematization of given facts or inner-mathematical argumentations. The following example of a task taken from a performance test in class 7 checks the mentioned competencies to be enhanced by CAS-support in math-lessons.

The task “mobile phone rate“

A one minute telephone conversation with a prepaid mobile phone costs 0,30 € during the business hours. The correlation between the number of minutes and the price has to be represented in three different ways: by a table of values, in the coordinate system, by means of an equation.

In addition to the presentation forms table and graph this task also requires the symbolic presentation of an equation. As the use of CAS in maths lessons is intended to improve the dealing with different presentation forms, it is expected that the learners become sufficiently familiar with them to be able to reproduce their acquired skills even without CAS-support. Given the fact that special properties of function classes are better memorized through permanent demonstration the pocket computer can help the learner to understand that the mobile rate represents a proportional correlation. Having to choose the settings of the screen layout for graphic presentations, the learners in CAS-supported lessons develop ideas of dimensions and

the behaviour of a graph. It is expected that this intuitive approach might help in the mathematization of the task "mobile rate" so that the learners have less problems to find axis scales and titles, all the more as they have already worked with variables, tables of values and corresponding graphs while dealing with CAS.

In the performance test of class level 8 the learners are allowed to use the CAS-instrument. The task was modified in a way that two rates are juxtaposed:

a) The phone rental tariff "Funny" is 6,49 € per month, a one minute conversation costs 10 Cent. Peter had 80 minutes of phone conversation in June. How much does he have to pay?

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b) The tariff "Happy"is only 4,99 € per month, a one minute conversation costs 12 Cent. Show the correlation between the number of minutes and the monthly amount of invoice in the following three ways: in a table of values, in a coordinate system and by an allocation specification.

c) Which monthly talk time makes a change from tariff "Happy" to tariff "Funny" profitable ?

d) The service provider wants to offer a third mobile phone rate "Lucky". A change from tariff "Happy" to the new tariff "Lucky" shall be profitable at a monthly talk time of more than one hour . What monthly rental and what price per minute would you suggest? Explain your proposal! While task a) is only testing the translation from everyday language into mathematical terminology, task b) requires that a mathematical problem is presented in different variations. In the present case the context is more complex than in the task of class level 7 because there is no proportional correlation. The presentation of the different forms requires the same competencies as in the task for level 7. In task c) a price comparison has to be made without defined solution path. The task can be solved either by setting up and solving of a linear equation or by extension of the graphic presentation in task b) or by compiling a table. It is possible to use the CAS for the three options, the graphic program or the table tool of the pocket computer. The test person is expected to plan the use of the maths tool, to manage the process and to document the results. Task d) requires imagination and creativity from the learners. They are supposed to find, by combining different informative elements and by trying, a new tariff which meets mathematical requirements. The solution of this task is not as clear as in a), b) and c) as the searched tariff can be fixed on the basis of different rentals and minute tariffs. Modelling competence and mathematical argumentation are required, following Kieran and Saldanha (2005) they are enhanced by CAS-supported maths lessons. In the documentation of the results the learners have to translate the computer language into mathematical terminology and everyday language which is one of the project targets of CAS-supported lessons. The presentation of the results by CAS in the lessons demands special training and practice (cf. Ball and Stacey (2003)). At the same time students are learning the structured and stringent presentation of a mathematical context.

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Training of mathematical skills without calculator

The goal of the CAliMERO project arouses the question which mathematical methods should be mastered by the learners without computer support. The whitebox-blackbox-principle introduced by Buchenberger (1989) deals with the question to what extent an automation is reasonable. In sustainable maths lessons the choice of the elements to be mastered in the sense of a whitebox has to be coherent with the taught subject and should not impede successful further learning. Studies by Ball, Pierce and Stacey (2003) show that learners who had been working with CAS-support in the lessons had problems to identify equivalent equations. The project tries to avoid those effects. Regular mental arithmetic exercise in elementary maths topics is part of the conceptual framework of the project. So-called first exercises with identification and realization activities are necessary to help learners deal with simple examples without calculator-support. The students learn to adopt an equalization method for the manual solving of linear equation systems while other procedures are solved following the blackbox-principle directly with calculator-support without having to be mastered by the learners.

A mental arithmetic test for the assessment of manual abilities represents an evaluation instrument which was run at the beginning of class 8. This 10 minutes test includes 14 items to examine mental arithmetic skills by basic tasks dealing with fractions and linear equations, simple geometric questions, dimensions and the calculation of areas and volumes. A test task example is: *2 cans of Coke cost 1,60 € at a kiosk. What is the price of five cans of Coke?*

PRESENTATION OF RESULTS

An analysis of the differences in the performance development experienced by the learners who participated in the CAliMERO project and the students from reference classes during the period between pre- and post-test in class level 7 did not show significant divergences in the test result as a whole. After the first teaching year this result was not unexpected because the learners had to become familiar with the CAS-instrument and the new teaching concept. A significant increase of student performance due to the school test is only expected at long term. However, certain test items revealed significant performance increases of the CAS-classes in comparison with classes using graphics calculators after only one school year.

Evaluation of the task “mobile phone rate“

The performance increase of learners using CAS in maths lessons amounts to 21,9% while the performance increase of learners who used gaphics calculators reached 10,4% in the course of the school year. This significant difference can be attributed to the use of CAS.

By means of the extended task in class level 8 the pre-test with CAS-use run at the beginning of the year shows a further performance increase for the three presentation types of phone rates in both groups. However, a processing of the creative task d) is

only documented by 10% of the learners. The only important aspect they concentrated on was to make sure that the tariff "Lucky" is cheaper than "Funny". The criterion “profitable at a monthly talk time of more than one hour” was often neglected. The learning results at the end of class level 8 are evaluated in June 2007 and will be available on our homepage from autumn 2007 (www.prolehre.de).

Feldfunktion geändert

Results of the mental arithmetic test

The results of the mental arithmetic test run at the beginning of the school year show that the integration of regular mental arithmetic exercises in the maths lessons strengthens the manual abilities of the learners. There were no significant differences between the students working with CAS and those of the compared classes, the learners still master the solving of simple equations and transformation of terms.Eye-catching problems only occurred in one task with multiplication of fractions (1/3 · 2/5) and in a task about the transformation of square measures (5cm² = _mm²), less than 50% of the test persons from both groups were able to solve the two items.

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Enhancement of low attainers

One of the most important observations in the evaluation of the performance test of the CAliMERO project run in class level 7 is the obvious improvement of low attainers between pre- and post-test. The learners were divided into three groups depending on their results in the pre-test with the development of their performance being examined after one year. The study LAU9 (2001) showed that a (normal) performance increase in maths lessons of around one third of the standard deviation can be expected during one school year. The comparison between the data calculated on the basis of this experience and the empirically measured values in the post-test shows on the one hand that students who had been rather low attainers in the pre-test and who were experiencing CAS-supported lessons performed much better than expected and, moreover, that they also performed better in the post-test than their schoolmates who were working with graphics calculators.Table 2 illustrates this result.

Attainment of group	Used calculator	Pre-test average + 1/3 standard deviation	Measured post-test average	Difference
below average	CAS (N = 129)	8,81	15,60	6,78
	GTR (N = 18)	9,20	13,06	3,85
average	CAS (N = 483)	18,01	20,82	2,81
	GTR (N = 90)	18,00	21,60	3,60
above average	CAS (N = 18)	28,16	29,78	1,62
	GTR (N = 6)	28,71	30,33	1,62

Table 1: Real and theoretical average post-test values with distinction of the performance groups (by test points, maximum 42 points)

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In 18 out of 26 test items the low attainers in the school test showed higher performance increase than the students from the reference classes. This performance increase of low attainers can mainly be observed in items with a high degree of algebraic argumentations. The low attainers also showed clear improvement in tasks requiring the interpretation of graphics, compared with the reference students. In the described task on the mobile phone rate the low attainers improved by 24%, the students from the reference group by 16%. Our interpretation of these results is that CAS-supported maths lessons, in combination with a new teaching culture which concentrates on the safeguarding of basic abilities, are most suitable to enhance the special development of the described competencies of low attainers.

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