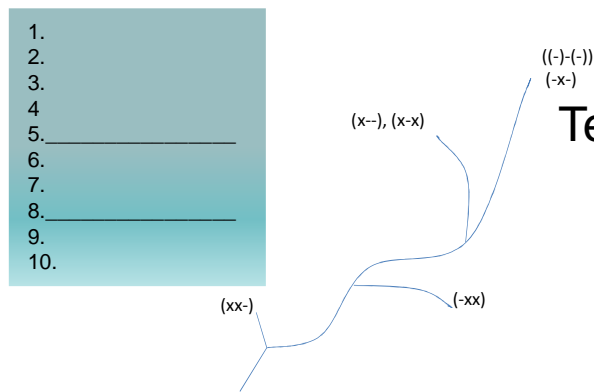


The Relationship Between Task Design and Sustained Competencies



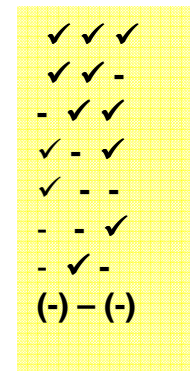
A TYPIIFICATION OF TASKS SUBJECT TO THE TARGET STRUCTURE AS BACKGROUND FOR LEARNING SURROUNDINGS IN VIEW OF THE SUSTAINED INCREASE OF COMPETENCIES



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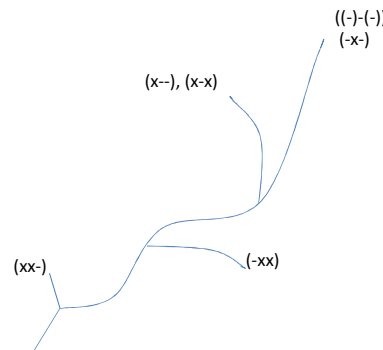
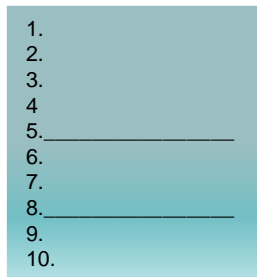
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PME 2010



Structure

- Background and the meaning of „Aufgabe“ (task)
- Which different structure types of tasks exist and how they enhance sustained learning?
- Learning environments based on the structure types of tasks: „set of tasks“ and „blossom tasks“



The meaning of „Aufgabe“ (task)



History:

Lenne (1969) deals critically with the *task didactics* – a limited notion of task

Dietz (1981) developed the *leading task concept* - according to Davydov (1977) included a hierarchically structured task system with far-reaching scope and graduated subtasks

In the nineties the notion of "*open-ended maths tasks*" led to an enlargement of the task properties.

Now:

The goals of modern math lessons are

- a higher individualisation of students' learning and
- the sustained increase of competencies.

How and which tasks can help?

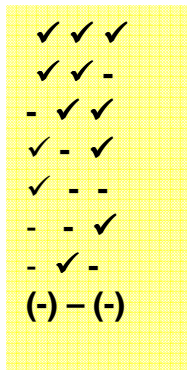
Which different structure types of tasks exist and how they enhance sustained learning?



The diversity of possible and required student activities in maths lessons makes an enlarged task notion seem advisable:

Following **Lompscher** et al (1985 – “*activity theory*”) we understand a task

- as an invitation to *develop learning activities* with the components



- these three components are known (✓) or unknown (-)

Every task can be described by one of eight possible triples for the target structure.

The notion of **task** has now been located above the notion of **problem**. **Problems** are considered as individually difficult **tasks**.



✓

✓

✓

solved problem, example, pattern

✓

✓

-

easy task („basic task“)

-

✓

✓

inverse easy task

Basic task:

-Simple mental arithmetic tasks: $12^2 = ?$

-Solve the quadratic equation $3x^2 + 7x = 8$ (the necessary approach/algorithm is known)

Inverse easy task:

- Give a quadratic equation with the solutions 3 and -2. Can you find more than one?



✓ ✓ ✓

solved problem

✓ ✓ -

easy task

- ✓ ✓

inverse easy task

✓ - ✓

proving task, a strategy finding task

A proving task:

Why is the formula applicable for quadratic equations $f(x) = x^2 + px + q$?

Or:

A strategy finding task:

Frank is always winning the NIMM-game. **How does he proceed?**

The NIMM-game is played as follows: There are 20 matches on the table. Two players are playing against each other. The player who can take the last match – if one, two or three matches per move are taken – will be the winner.

Initial situation	Transformation	Final situation	
✓	✓	✓	solved problem
✓	✓	-	easy task
-	✓	✓	inverse easy task
✓	-	✓	proving task, a strategy finding task
✓	-	-	(difficult) determination task
-	-	✓	(difficult) inverse determination task

Is the tetrapack milk carton for 1 l an optimal packaging?

A pond is planned with a surface of 10m². What will be the dimensions of the pond?

Initial situation	Transformation	Final situation	
✓	✓	✓	solved problem
✓	✓	-	easy task
-	✓	✓	inverse easy task
✓	-	✓	proving task, a strategy finding task
✓	-	-	difficult task
-	-	✓	difficult inverse task
-	✓	-	a request to invent a task

Write a story including the three key questions of percentage calculation !

Initial situation	Transformation	Final situation	
✓	✓	✓	solved problem
✓	✓	-	easy task
-	✓	✓	inverse easy task
✓	-	✓	proving task, a strategy finding task
✓	-	-	difficult task
-	-	✓	difficult inverse task
-	✓	-	a request to invent a task
(-)	-	(-)	modelling task

How long does it take to change the water in the swimming pool?

Or:

Make a survey with your fellow students and present the results!

A useful theory...



-
- On the basis of many years of experience in the teacher training and teacher further training we can say that the orientation on these eight structure types leads to greater methodical variety in the planning of teaching units, and offers students the possibility to understand the learning content far beyond formal reproduction and application.
-

A useful theory...

-
- On the basis of many years of experience in the teacher training and teacher further training we can say that the orientation on these eight structure types leads to greater methodical variety in the planning of teaching units, and offers students the possibility to understand the learning content far beyond formal reproduction and application.
 - These 8 structure types may serve as a guideline and construction heuristics for the design of units for self-directed learning, learning by stages, expert puzzles or for the provision of learning conditions to stimulate cognitive activation.
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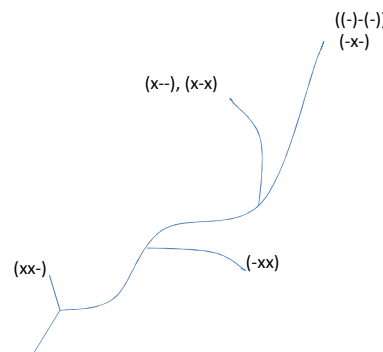
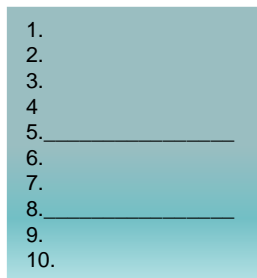
A useful theory...



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- On the basis of many years of experience in the teacher training and teacher further training we can say that the orientation on these eight structure types leads to greater methodical variety in the planning of teaching units, and offers students the possibility to understand the learning content far beyond formal reproduction and application.
 - These 8 structure types may serve as a guideline and construction heuristics for the design of units for self-directed learning, learning by stages, expert puzzles or for the provision of learning conditions to stimulate cognitive activation.
 - The structure types of tasks are useful to analyse
 - learning environments in textbooks and find gaps in the diversity of tasks.
 - artefacts of teacher trainings or the quality of e-learning environments.
-

Structure

- The meaning of „Aufgabe“ (task)
- Which different structure types of tasks exist and how they enhance sustained learning?
- Learning environments based on the structure types of tasks: „set of tasks“ and „blossom tasks“



Set of tasks for the *first exercises* to calculate a zero of linear functions



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Choose five tasks and you have 15min to complete them.

Find the point, where these linear functions cross the x-axis (zero): (✓ ✓ -)

1. $f(x) = x - 5$

2. $f(x) = 2x + 6$

3. $f(x) = -5x - 2.5$

(- ✓ ✓)

4. Draw a Graph of a linear function with the zero at $x = -5$

(✓ - -)

5. What are possible applications of a zero of a function? (What could the zero mean?)

Task-set for the *first exercises* to calculate a zero of linear functions



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Choose five tasks and you have 15min to complete them.

Find the point, where these linear functions cross the x-axis (zero): (✓ ✓ -)

1. $f(x) = x - 5$

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(- ✓ ✓)

4. Draw a Graph of a linear function with the zero at $x = -3$

(✓ - -)

5. What are possible applications of a zero of a function? (What could the zero mean?)

6. Find the equation of two linear functions with a zero at $x = 4$

(- - ✓)

7. Find an equation of a linear function without any zeroes.

(- ✓ -)

8. Find a context which can be described with a linear function with a zero at $P(1,0)$.

Task-set for the *first exercises* to calculate a zero of linear functions



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8. Find a context which can be described with a linear function with a zero at $P(1,0)$.

9. Why do linear functions not have more than one zero? (✓ - ✓)


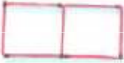

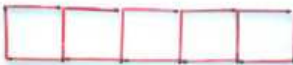
10. Find an expression for the determination of the zero of $f(x) = mx + b$ and give the necessary condition for m , x and b . ((✓) - -)

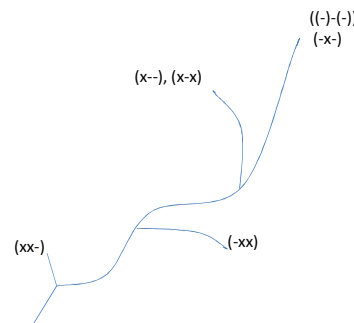
Blossom task as self-differentiating exercise form

Work 15 minutes on these tasks at choice:

Square chains have been made with matches

a) Complete the table. (✓ ✓ -)

	Number of squares	Number of matches
	1	4
	2	7
	3	10
?	?	?
	5	16



b) How many squares would 49 matches make? (- ✓ ✓)

c) Find an expression for the number of matches, if q is the number of the squares. (✓ - -)

d) Create another pattern with matches and find the expression for this situation. ((-) - -)

Optional choice strengthens the self-responsibility of the students for their learning and gives a feeling of success

The same **context related** to all subtasks simplifies the discussion of the tasks

Low-threshold access makes the task accessible also for low attainers

Different task types allow individual and successive development of learning contents

Open subtasks enhance problem solving competencies and the creativity of students and support different levels of processing

Download presentation: www.math-learning.com

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Now there is an agreement in Germany on what mathematical education at school should be able to provide for all:

“Maths lessons contribute to general education by providing three essential experiences:

(G1) to discover natural, social and cultural phenomena in the world which affect us - or should affect us - in a specific way and to understand them,

(G2) to get to know and recognize mathematical objects and facts, represented by language, symbols, pictures and formulae, as mental creations and as a deductive world sui generis,

(G3) to acquire, by dealing with tasks, problem solving competences (heuristic skills) beyond mathematics.” Winter (1995, p. 37)

Tasks can take on *functions* on several levels:

- tasks to develop or control *intelligent knowledge* (terms, theorems, approaches, strategies, application fields),
 - tasks to obtain, enhance or identify *competencies to act* (recognition, description, connection, application, explanation of mathematical terms, correlations or proceedings),
 - tasks for the development and description of *meta competencies* (among others development of an adequate mathematical view, enhancement of self-assessment).
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