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Intelligent and reflected maths exercising – the didactic quality of teaching and learning material

Various exercises in maths lessons and homework are intended to enhance the individual mathematic and self-regulatory competencies of the students. The presented list of criteria for the quality assessment of mathematical exercises includes valuable aspects of teaching and learning material.

The list of criteria shows that tasks are the main tool of maths teachers especially for exercises. In this article we would like to present quality criteria for intelligent and reflected exercising which are based on didactic knowledge and experience and show the application of such criteria.

What are quality criteria for teaching and learning material good for?

In further teacher training courses which are run by us within the framework of a learning platform (www.proLehre.de) over 6 months and which are intended to support teachers in the testing of didactic concepts, the participants develop their own material according to the recommendations in the further training courses, gather experience and report on it. Our task is to give the teachers feedback regarding their material. Manageable, flexible criteria were required as an orientation guide to make the aims of the lessons transparent and operational, without creating a rigid pattern.

A list of criteria in the form of an orientation guide which allows the assessment of learning material is helpful for self-reflection on the learning material used and reveals the potential for development. Moreover such a list of criteria allows the description of the effects of the teacher training and further training.

However, there are also reservations about the application of lists of criteria for the quality assessment because so far no list has been elaborate enough to take into consideration all the actual conditions of a learning group and also because it is, in contrast to other areas of society, rather unusual for a professional group of teachers to deal with such questions. To give an example: when buying learning software we are not sure if it is „good“ or „bad“ and sometimes there is a negative or a positive surprise, cf. the studies in the framework of a seal of quality for computer supported learning arrangements by Bruder/Brücher/Sonnberger (2006).

The expectation that the learning materials for our children at school are also “good“ is justified and therefore we should harmonize what we want to be understood as “good“ (because quality is normally the result of a harmonization of good criteria) and describe it.

We consider our approach as an instrument to help teachers develop their teaching possibilities. However, the course of possible further development is determined by the teachers themselves, and these criteria are just an offer of assistance.

Where do quality criteria for exercise materials come from?

Quality criteria for didactically valuable exercises can be found in current research results, in articles about the further development of maths lessons and in meta-analyses about the quality of lessons (cf. among others, Leuders, 2001; Büchter and Leuders, 2005; Helmke, 2003; Blum

et al., 2006; Jordan et al., 2006). The aspects of maths exercises or teaching and learning material described in the literature are mainly focussed on specific aspects of tasks, like openness, motivation or everyday life situations. Missing, so far, is a global view on teaching and learning materials, which includes all relevant aspects and is adapted to the teaching routine.

On the basis of conceptions, which are considered established in didactics, of didactically valuable math tasks and high-quality lessons, we collected and developed criteria for the assessment of teaching and learning material on the basis of specific work products. The criteria presented describe a horizon of expectation for tasks in the sense of a required quality. Naturally it is not possible to take up in every single task all criteria which are desirable in terms of a general view of quality lessons! Even a whole task series will be focused on specific aims and not suited to cover all didactic options at the same time. However, the present list of quality criteria offers an orientation for the personal development of exercise material from a general point of view. It also allows the recognition of possible divergences in the offer of tasks (e.g. levels of difficulty), missing aspects (e.g. options) and whether important aspects have „instinctively“ been implemented. It is already available for individual tasks, work sheets, long-term homework and also for written tests and complete teaching plans for lessons.

In the following table the criteria taken from the literature and developed for the assessment of learning material for intelligent and reflected exercising are presented in the form of a check list, see box 1. Box 2 describes how to use this check list for the assessment of specific exercise material.

Category	Criteria	Implementation	
		☺	Development potential
Subject-specific background	Subject-specific adequacy of exercises and correctness of sample solutions		
	Subject-specific adequacy of the tasks		
Transparency	Clearness of tasks and of the horizon of expectation		
Motivation	Adequacy of the tasks and contexts (age-appropriate, language, reference to sense and subject, consideration of the learning group...)		
	Design of learning material (logical composition, legibility…)		
Starting level	Conception and consideration of the starting level (diagnostic elements)		
	Repetition offers to close individual gaps		
Internal differentiation	Consideration of individual need for exercises with respect to content and time (tasks with different degrees of difficulty, graded series of tasks, additional tasks, optional tasks)		
	Open tasks (Blossom and funnel tasks, solution variety, inventing of solutions)		
	Adequate social forms (individual-, partner- or group work)		
	Addressing of different levels of perception		
	Differentiation of the contexts depending on the main focus of interest		
Cognitive acti-	Internal cross-linking in the sense of intelligent exercising		

Category	Criteria	Implementation	
		☺	Development potential
vation	Potential for the use or creation of heuristic solution strategies		
	Variation of the "target types" for tasks (basic tasks, inversions, extensions)		
	Variation of learning activities (understanding, describing, performing, interlinking, explaining, interpreting)		
Self-regulation	Offer of support for the planning of activities (time schedule, target planning, use of strategies)		
	Offer of support for reflection (solution control, approach, control of the own learning success)		
Use of teaching aids	Use of technical teaching aids (adequacy, purpose)		

Box 1: list of criteria for the review of learning and teaching material for intelligent and reflected maths exercising

How to use the list of criteria for the assessment of learning material

Choose one of the learning materials you want to use in your lessons, e.g. a work sheet to practise new learning content. Try to find out which aspects of a given criterion in the list might correspond to your learning and teaching material and to your targets, and try to evaluate its quality. Tick ☺ if the implementation of the criterion meets your expectations. If you see more possibilities for development, make a note in the neighbouring column. A glimpse at the table quickly reveals what can be maintained when the next learning material is selected and what could be added as selection criterion.

The categories and criteria listed in box 1 are explained below. We believe that all categories are relevant to some degree for the teaching and learning material for intelligent and reflected exercising. Regarding the criteria it is possible that, depending on the objective of the exercise, specific criteria are not or almost not relevant. A flexible choice of the criteria is therefore recommended.

- The category *subject-specific background* is one of the obvious elements of assessment. This also depends on the objective as naturally it is also reasonable to provide a task with an incorrect solution and to let the students find out the mistakes. The *Subject-specific adequacy* describes whether the specific context corresponds to reality if this is suggested. This criterion should generally be used to see if the tasks within the teaching and learning material support successful further learning or if it might be impeded by too narrow schematisation or the support of inappropriate basic ideas.

- The criterion *transparency* requires clear targets in the teaching and learning materials for the students. This applies to the objectives of the tasks as well as to ideas about the

horizon of expectation especially for open tasks. The horizon of expectation is not related to the result or a certain solution process but to the frame conditions of the solution process. It is necessary that the students know that solution methods have to be taken down, assumptions documented and results interpreted or explained and what strategies can be adopted. The learners should not have to guess what the teacher is asking for! It is also necessary that from the teacher's perspective the task and the targets of competency, the subject, the didactic functions of the material used and structuring elements like the processing time for homework, must be clear.

- From the *motivational* point of view the *relevance to age of the task and the context* related to questions and exercises (e.g. riddles, resolving contradictions, finding out mistakes etc.), which make sense with or without respect to maths, should have a comprehensible relation to the subject. The motivation potential of a task can also be increased by assigning an expert role to the students, e.g. „Advise on the following decisions", „Who is right?", „Can you help?". In contexts related to reality the authenticity is very important for motivation (cf. Leuders, 2001). The *design aspect* aims at a well structured, easily understandable and logical structure of the learning material. Particularly in learning surroundings for independent practice, appropriate illustrations can help understand the tasks and realise the objective of the exercises. A well-elaborated, subject-related visual design of the learning material makes learners feel they are taken seriously and encourages careful processing of the tasks.

- The criterion of consideration of the *starting level* intends to find out to what extent the level of attainment and development of the students is considered in the learning material. These might be diagnostic elements or modules for the self-assessment of the students or offers to close possible knowledge gaps. *Regular repetitions* to consolidate what has been learned, e.g. in the form of mixed mental arithmetic (see the article in this issue) are among the more didactically valuable elements in the preparation of a teaching unit.

- *Internal differentiation* is a central quality criterion of good teaching (cf. Helmke, 2003; Leuders, 2001; Büchter and Leuders, 2005). First of all it is necessary to take into account the students' individual need for practice. Varied tasks with different levels of difficulty are suitable for this purpose, for example task sequences with graded levels of difficulty (cf. Reibis 1996), optional tasks (also for homework) or additional tasks. *Open tasks* (e.g. blossom and funnel tasks, tasks with varying solution methods, inventing of tasks) have a particular potential for internal differentiation. Moreover a *differentiation of the contexts* can be useful to offer the students who are interested e.g. in football or other sports the possibility of identification with the specific mathematical subject even if complex variations of the subject will be exceptional. Internal differentiation also implies the consideration of different learning approaches and different levels of understanding (cf. Zech 2002) – enactive, iconic, and symbolic. The choice of a suitable social form is especially interesting for differentiated exercises: it allows special training by explaining and inquiring processes in partner or group work.

- In the category *cognitive activation* the exercises are especially focused on different *variations of the degree of difficulty* and the interlinking of learning contents. Intelligent practising implies a particular variation of specification requirements, which is reflected in the use of different task formats and in the type and variability of the required learning activities. For intelligent practising the task types: basic task, modification of a basic task (especially inversion) and extension of a basic task are expected.

- *Learning activities* for cognitive activation are concentrated on elementary student activities like recognizing, describing, performing, interlinking, explaining, interpreting. Due to their different requirements, sustainable learning effects are only reached in combination within a learning environment. The criterion *variation of learning activities* for cognitive activation is connected with the expectation that they are also taken up in the teaching and learning material. Intelligent exercising may mean that different representational? forms of functional correlations can be recognized, described, performed, interlinked and possibly interchanged, their use explained and the results interpreted. The process-related competencies and levels of requirement described in the education standards (KMK, 2003) represent a very complex construct with global orientation which is less suited to evaluate the activation potential of specific exercises.

- In contrast to purely schematic exercising, reflected exercising is not possible without *offers of support for the planning and processing of activities* (cf. Schmitz, 2001), e.g. offers of time schedules, target planning, use of strategies. Students need opportunities to learn how to set realistic objectives, to manage the available time schedule and also to use, in addition to subject-related learning strategies, strategies for the improvement of concentration and the avoidance of distraction. S*upport offers for reflection* apply to solution control, approach, control of personal learning success, e.g. by means of a learning protocol, see examples below.

- If *technical teaching aids* are used in the learning material for exercises the function of these tools should be explained, e.g. the question whether a special tool should be adopted or if a DGS or CAS should reasonably be applied as a control for a solution found without the use of teaching aids. Basically the adequacy and purpose of the use of technical aids is considered, starting with the use of plotters, and ending with the use of computers or the internet.

Example for the evaluation of a learning protocol with the list of criteria

A learning protocol is one of the special methods for explicit reflected exercising, cf. Bruder (2007). In order to fulfil its function, the learning protocol should meet the following requirements:

- Limitation to central theme of a subject, limitation of the time frame (15-30 min) and individual processing by the students to allow self-assessment of the learning level
- Comparison of a basic and an inversion task on a specific subject for targeted, intelligible cognitive activation
- Establishing of an everyday reference or typical application of the subject

- Gathering of process-related aspects of lessons like “how did we proceed?“ or discussion of the source of typical mistakes.

The learning protocol below (box 3) was used in the framework of a sequence of 10 lessons on the “Calculation of surface - strategy: decomposition and reduction to known elements”. The subject of the lessons in class 7 was the calculation of surface of triangles and rhomboids. The learning protocol was used in the 3rd lesson of the sequence and presented on a teacher further training course.

Learning Protocol

Subject: Calculation of surface

1. A square has a side length of a=4dm. Calculate surface area and volume.
2. Specify the possible dimensions of a rhomboid with the surface area 12m².
3. If the triangle is divided in half, the surface area _____.
4. Describe a (extra-curricular) situation where the calculation of triangular areas could be helpful.
5. Explain the strategy for the formula derivation of the rhomboid area.

Box 3: Example for a learning protocol (class 7)

The requirements for a learning protocol described above were well adopted. There is a clear and flexible focus on the calculations of surface without completely leaving out the perimeter as “counter-example” because the distinction of both terms causes normally problems.

The categories are assessed as follows:

The tasks in the learning protocol are generally correct; in task 3 a generalization for all triangles would be recommended. The request to explain “the” strategy for the formula derivation of the rhomboid refers to “the” reduction strategy used in the lessons. However, as there are, within the reduction principle, several interesting partial strategies to determine the surface area of a rhomboid (decomposing into one rectangle and two congruent triangles, completing a rectangle of equal area), different answers may occur. (Note: If only the realisation of the reduction principle was covered in the lessons there might have been a waste of potential for cognitive activation and internal differentiation in the lessons before) In task 5 the students might be allowed to choose the strategy they wish to explain. There is a given transparency of target for the students connected with the learning protocol. The tasks are clearly and accurately formulated and the function of the learning protocol was explained to the students.

The preparedness to process on the learning protocol is supported by short and intelligible questions which are closely connected to the lessons. Moreover the demand to give examples from everyday life to show where the calculation of surface areas might occur helps to develop their own conceptions of maths applications, which makes the students feel taken seriously. There are no specific visual features which is not exactly a disadvantage in this case. Basic knowledge of square measures and units of area are indirectly expected. Internal differentiation is supported by open question 4, the potential of task 5 was already discussed above. It is eyecatching that actions are generally quite abstract and visualizations left out which

would however be advisable in task 2, in order to vary the extent of abstraction within the tasks and to reduce the requirement leap between task 1 and task 2. Due to a variation of the level of difficulty and different student activities (e.g. proceeding, interlinking, explaining) this learning protocol has high cognitive activation potential. It might be favourable to explicitly treat typical sources of mistakes in the calculation of a rhomboid, but this could also be included in the discussion of task 2.

The learning protocol has self-regulation potential beause the students have the possibility to assess their learning level and to reflect on learning strategies which have been relevant in the previous lessons. The use of tools was not relevant in the learning protocol.

Outlook

Teaching and learning materials can be analysed under different aspects. The list of criteria presented above does not claim to be complete with respect to the relevant quality criteria but to manageability and flexibility because the specific purposes of the materials represent the central input parameter for evaluation. Even if exercising as didactic function of the materials to be evaluated was in the center of interest it is obvious that the categories in the left column may also apply to materials with other purposes. However, the criteria would have to be differentiated further.
A list of criteria for the assessment of learning material is helpful for the self-reflection about used learning materials and has potential of development. It is taken as a basis for the feedback of work products of the participants of our teacher further training courses. Therefore the presented list of criteria allows to describe effects in the teacher training and further training. It is by no means trivial to generate analogous examples also for other subjects (e.g. a learning protocol).
Of course the evaluation of teaching and learning material is limited regarding the influence on the lessons development. The effects of real lessons do not depend on the learning potential of the teaching materials but on what is made of it!

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