CAS USE IN SECONDARY SCHOOL MATHEMATICS - TEACHING STYLE AND MATHEMATICAL ACHIEVEMENT

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TEXAS INSTRUMENTS

Fachbereich Mathematik

Research interest in long-term studies about CAS-aided teaching

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- the development of mathematical achievement (gauged on the syllabus introduced)
- the perception of CAS-aided mathematics teaching on the part of the pupils, as well as the acceptance of the jointly-developed teaching concept by the participating teachers.

Answers were sought to the following questions:

- How does mathematical achievement change in CAS-aided teaching?
- Are there links between the actual teaching method and mathematical achievement?
- How do pupils' views on mathematics, teaching, and technology change?

Structure

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1. About the potential of CAS in the teaching and learning of mathematics

2. Targets of the project CAliMERO

3. The teaching concept of the project CAliMERO

4. Development of the student performance after the implementation of the concept by the teachers – some results

1. What do we already know about the potential of CAS/GC in the teaching and learning of mathematics?

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- **Reduction** of schematic processes (release of cognitive load)
- Supporting the **discovery** of mathematical issues
- Supporting **individual** preferences and approach
- Promoting the **comprehension** of mathematical correlations
- Supporting the need for security by providing **control possibilities**.

(e.g. Barzel 2011)

Investigations to the certain integral

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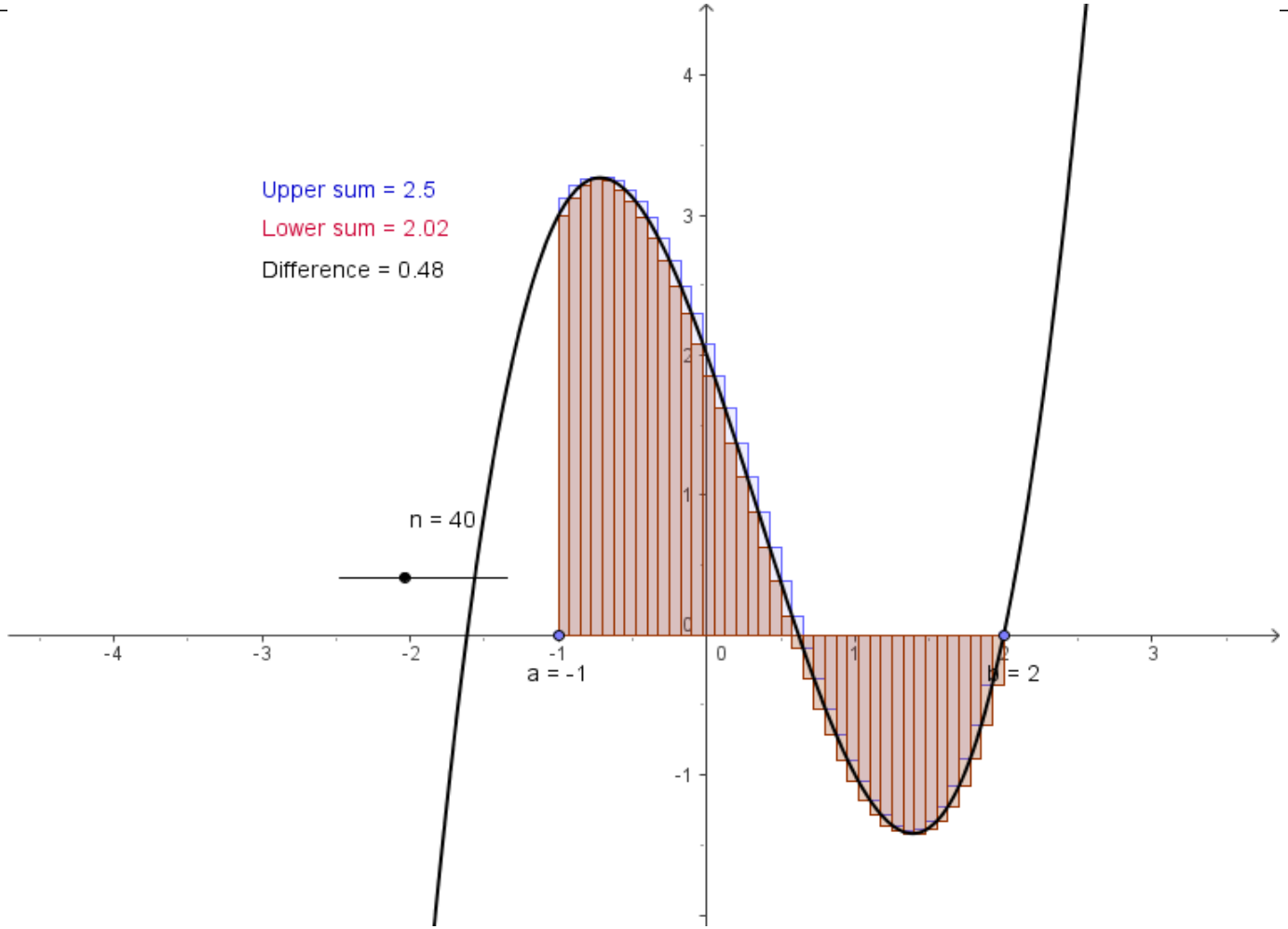

Use CAS/GC for understanding math

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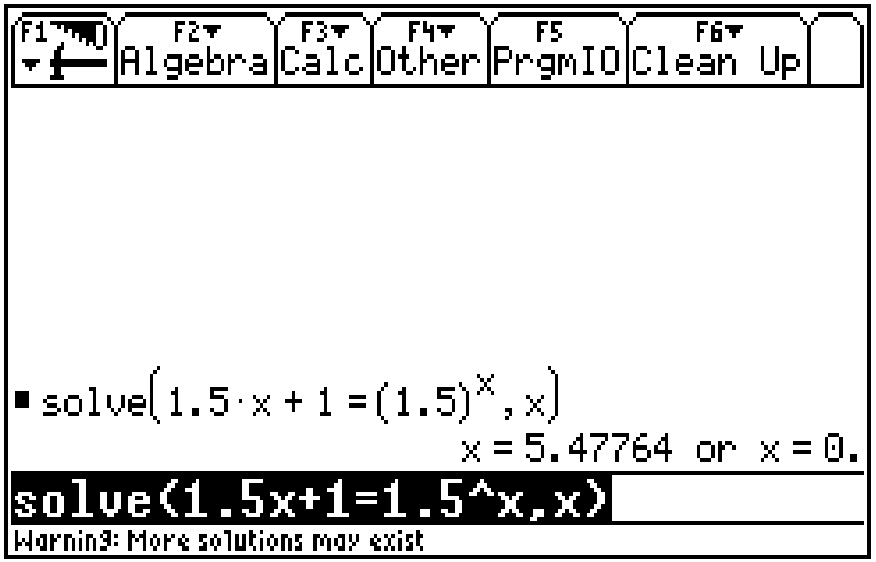
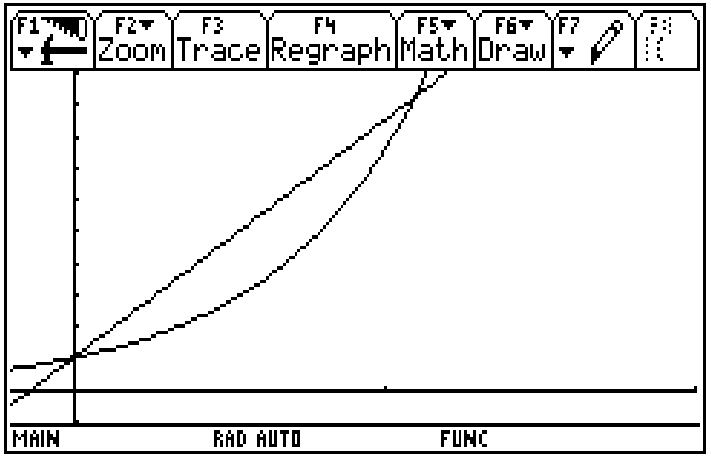
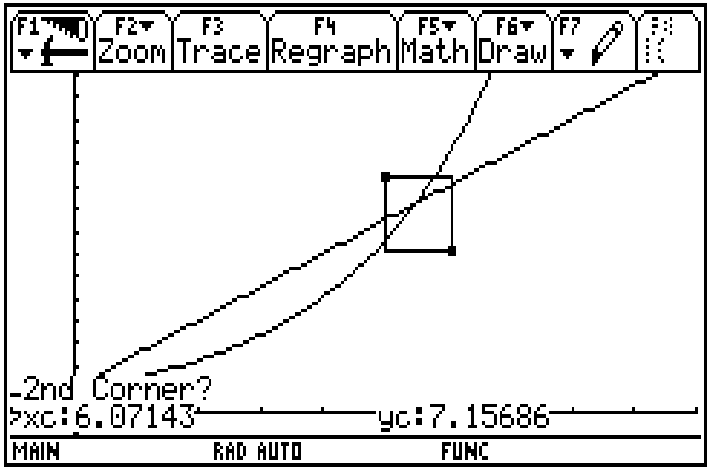
Given equation: $1{,}5x+1=1{,}5^x$







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1. What do we already know about the potential of CAS/GC in the teaching and learning of mathematics?

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Literature on the potential of CAS or graphic calculater (GC) in teaching reports:

- the potential of technology use in understanding mathematics
- control instrument and
- an occasion for deliberation (for example when developing hypotheses or clarifying the information on the display)

a) Make Otto for zero

(Guido Pinkernell, CAliMERO)

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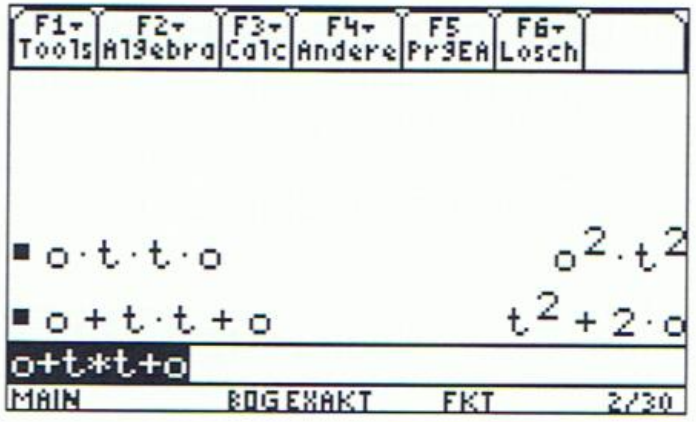

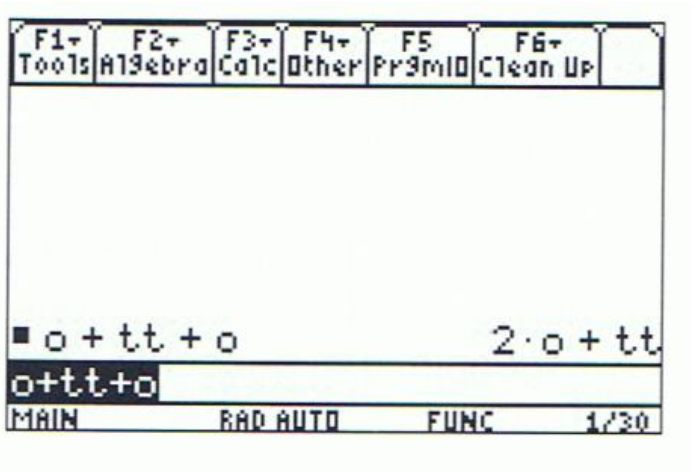

b) Make with Hannah the following terms: $a^2 \cdot h^2$ $h^2 + a^2 - n^2$ $n^2 + 2a$.

c) Make Hannah for zero. It is possible, to construct 1 or 2 with Hannah?

d) Describe the rules, used in b).

e) Which name is this? $\frac{s^2}{u} + 2a - u^2$

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Some consequences from the use of CAS on secondary level:

- consequences for curricular alteration in the light of the technical possibilities (such as translating between forms of representation of functional relations gaining in importance)

- need to clarify what should still be possible able with a handheld computer (i.e. manual capabilities and skills)

- transformations of terms must continue to be learnt and practised

- an aid-free section of the central examination

Part of a test for maths-students (Darmstadt University of Technology)

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Simplify!

$$\sqrt{\left(\frac{a}{2}\right)^2 + a^2} \quad ; a \geq 0$$

6 German federal states will include an aid-free section in the Abitur 2014

$$\frac{x^{m-n}}{y^{2n}} : \frac{x^{2m-n}}{y^{n-1}}$$

Find the equation of a straight line by both points P(3;5) and Q(-1;1).

How can a handheld calculator (HC) be used in a task?

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0 - HC not allowed or not reasonable .

1 - HC assumes control function for simple calculations or justifications.
2 - HC use reduces formal calculation or construction effort; the task is still solvable without HC .

3 - Calculator supports experimental situations, checking of assumptions etc.
4 - the task is no longer (effectively) solvable without HC, due to the quantity of data or the complexity of modelling.

5 - new mathematical correlations are explored using the HC.

Practice exercise 18 (problem solving) (Calimero, Band 5, p. 26)

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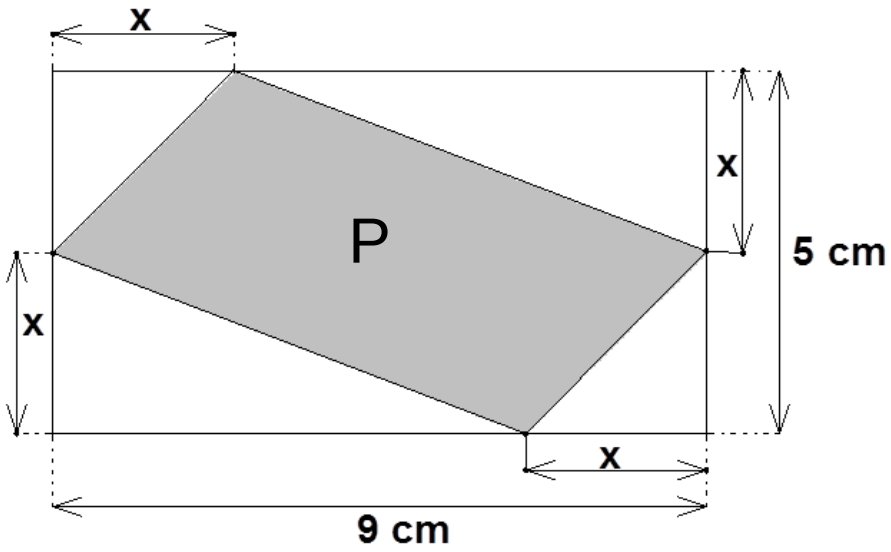
Parallelogram P is placed in a rectangle with side lengths of 9 and 5 cm, so that distance x is created clockwise from the corners of the rectangle.



Task:

Determine the length of x so that the area of the parallelogram is as small as possible.

Tip: you will obtain the area of the parallelogram when you subtract the area of the four triangles from the area of the rectangle.

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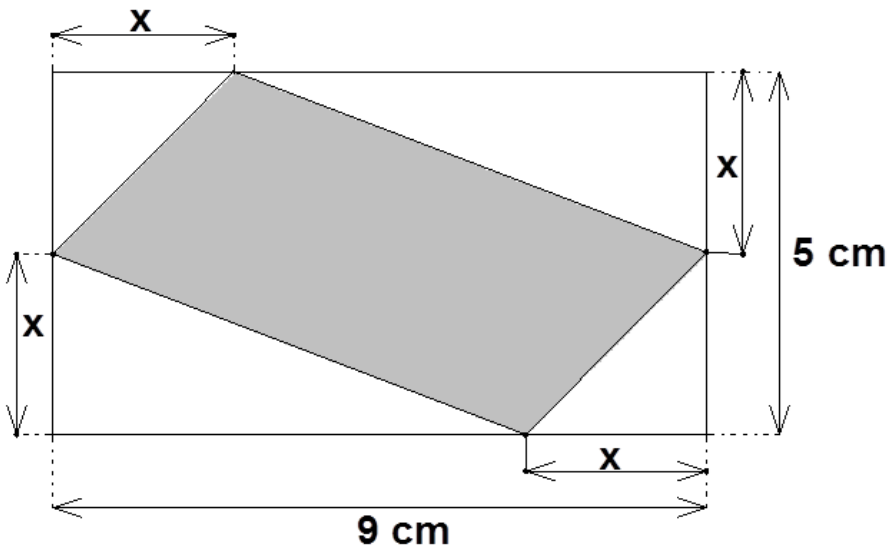
Purely algebraic solution approach:

P



Find a term to determine the area:

$45-(x(5-x)+x(9-x))=45-(5x-x^2+9x-x^2) = 2x^2-14x+45$

Recognise and plot a parabola => determine extremes => interpret

Processing the open assignment „by experiment“

P

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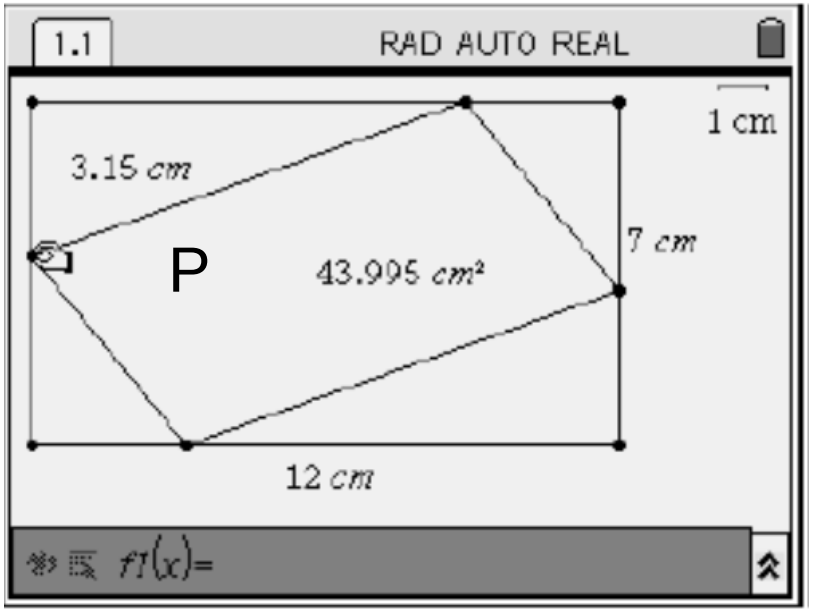
Open ended task: Investigate the area of the parallelogram P.



The construction of the figure is given by the teacher.

Observe the change in the area of the parallelogram upon changing variable x.
Record observations in writing => suggest a solution.

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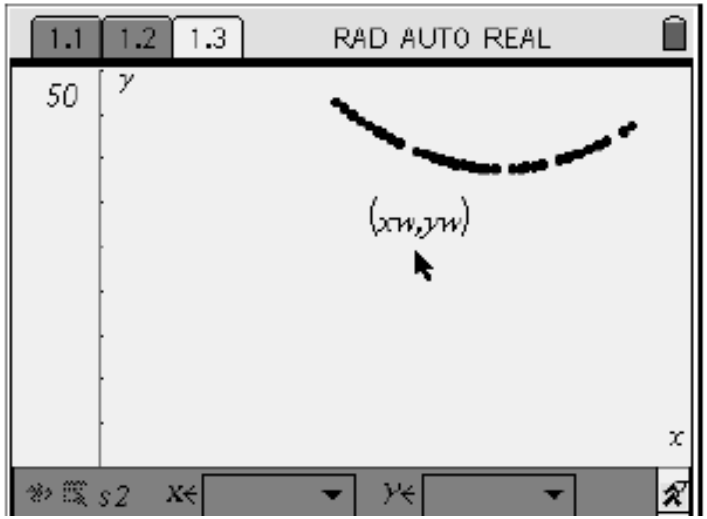
Pupils can save the two dimensions x and fl(x) and enter them in a table

⇒present the data points be presented graphically

⇒recognise that the points lie on parabola

⇒determine the lowest point of the parabola



A paradigm shift has taken place in the visions of the use of digital tools in mathematics teaching...

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20 and 10 years ago:
It was clearly not only a matter of developing and adapting the learning processes but particularly the educational content.

Nowadays,
calculator use and calculator potential are described against a backdrop of variable teaching and learning for promoting individual comprehension of fundamental mathematical relationships,
rather than with the aim of opening up completely new learning content.

Teacher demonstration and pupil exploration are mutually constructive and complement each other.

Structure

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2. The project CAliMERO on the CAS use from grade 7 (2005-2010-2013)

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CAliMERO:
Computeralgebra im Mathematikunterricht:
Entdecken, Rechnen, Organisieren

2005 - 2010:
29 classes on 6 schools in Lower Saxony,
supported by Texas Instruments

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The objectives of the first phase of the project were...

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-to investigate the use of a handheld CAS system in German secondary maths education from grade 7 to grade 10,

-with the aim of implementing and evaluating a teaching concept that focuses on sustainable learning of mathematical competencies and basic knowledge,

- while the evaluation concentrates on the pupils' mathematical performance, teaching methods, and the pupils' and teachers' views on mathematics, teaching, and technology.

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Research results on e-learning :

“the use of digital media does not automatically lead to better results than conventional educational concepts. On the contrary: their unreflected use is often leading to low acceptance by the learners, poor learning success and efficiency.“
(Kerres 2001, p.85).

3. The teaching concept of the project CALiMERO

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Tasks are in the centre - with the three components:

✓ ✓ ✓
✓ ✓ -
- ✓ ✓
✓ - ✓
✓ - -
- - ✓
- ✓ -
(-) – (-)

Initial situation

Transformation

Final situation

- these three components are known (✓) or unknown (-)

Task classification is an excellent method for the teacher to check the variety or top-heaviness of his own teaching with respect to the tasks chosen.
Changing perspectives on mathematical learning contents in different contexts allow and require exactly those cognitive activities which are necessary for understanding and cross-linked learning.

3. The teaching concept of the project CAliMERO

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- Learning materials were developed in regular meetings together with the participating teachers.

- These learning materials were intended for the multiple use of HC to support the learning process by taking into account the task variety described above.

- This development of tasks was structured by *typical teaching situations*, i.e. phases with a predominant didactic function in the lessons as follows:

-securing of the initial level for the progress of understanding and exercise,
-introduction of new learning contents,
-design of exercise processes and
-systematisation of mathematical learning contents and their applications.

(cf. Steinhöfel, Reichold & Frenzel 1985,)

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3. The teaching concept of the project CALiMERO

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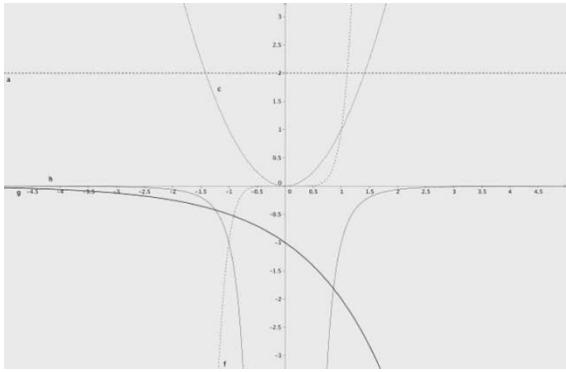
Example of a learning journal (power functions)

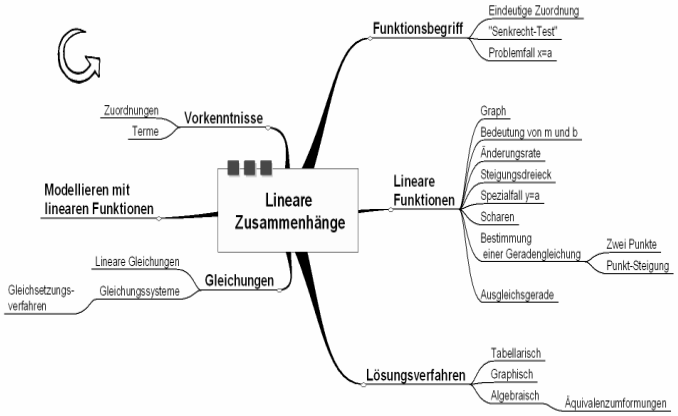


1) In the following figure you can see the graphs of the different functions. Decide which of the functions are power functions.

2) Determine the equation of a power function $f(x) = ax^b$ which crosses the following points:
P(-1,5/ -22,78) and Q(2,5/ 292,97).

3) Decide (with reasons!) which of the following situations can be modelled by a power function
a) The volume of a cube depends on its edge length a.
b) Paul‘s savings grow by 4% every year.
c) The area of a figure depends on the scale factor.

4) Lisa has entered the function $f(x)=x^{1/3}$ in her HC and is puzzled that the graph is a straight line.

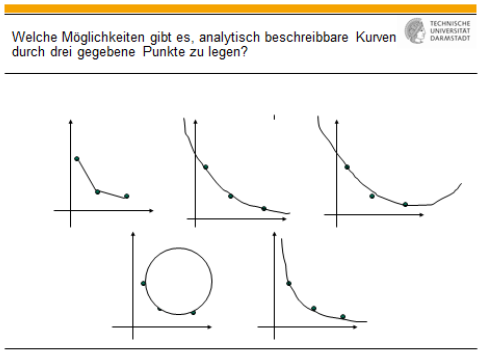


Check list

I can differ between linear and non linear functions ….
I can find the zero of a linear function.
I can give an equation of a linear function (2 points given).

Regular mental maths: 10 min for 10 items

Aim: to keep alive mathematical knowledge from previous learning contents which were already understood and which are essential for further learning.
- weekly or all to weeks on the same day in the same lesson (as a ritual)



Examples:

a) 0 = (a + 3) (a - 4)
b) 1:500.000 4cm on a landscape are in nature….?
c) 25% of 1hour

c) a = b ● c which context has this structure?
d) Coordinates of 3 points are given – which possibilities for a curve/function?

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Evaluating instruments

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- 29 project-classes from 6 schools (CAS)
- 6 control-classes (GC)

Evaluating instruments in the classes 7 to 10:

- **maths performance test** every year (pre-post)
- **student interview** (from class 8 with aspects of technical aid)
- an annually adapted **basic -knowledge test** (without technical aid)
- a **standardised lesson journal** (used to log the teaching activities with which the pupils gave information about the subject, tasks and the methodological structure of the lesson).

Download: http://www3.mathematik.tu-darmstadt.de/ags/didaktik/forschung/didaktik/projekte.

Performance test and basic-test (no HC)

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- 45-minute **performance tests** at the start and end of every school year
- Tasks with changing degrees of difficulty were set on basic mathematical contents and competence areas were created in an undulating open-ended design.
- A task- and test design of this kind can measure whether more tasks will be mastered (successfully) at the end of the school year, whether the type of solution approach chosen changes and the solution strategies become more diverse.

See Collet (2009).

- 15min **basic test** at the start and end of every school year 8,9,10 (without HC)

Summary of performance development
CAliMERO

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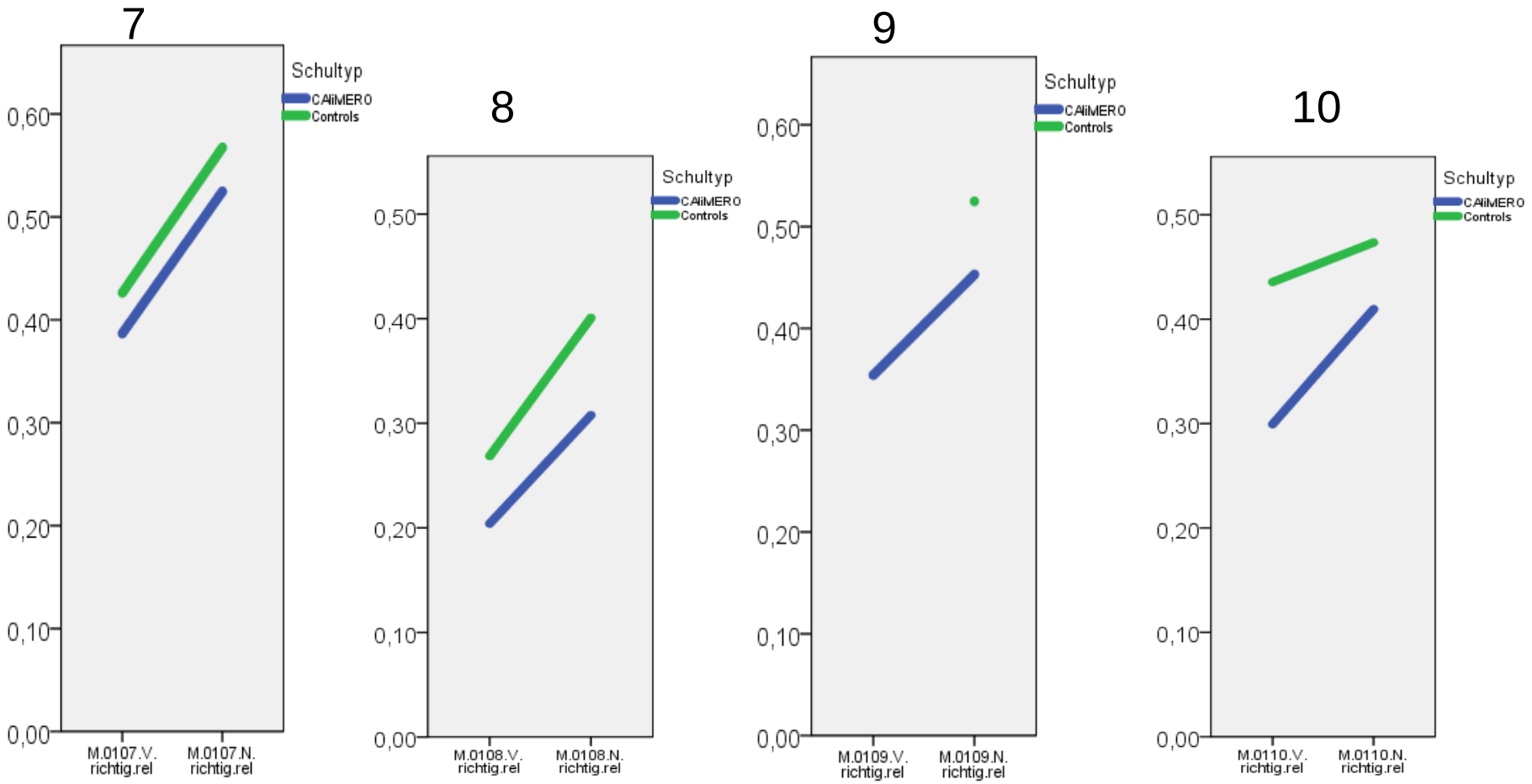

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Summary of basic knowledge development — CAliMERO

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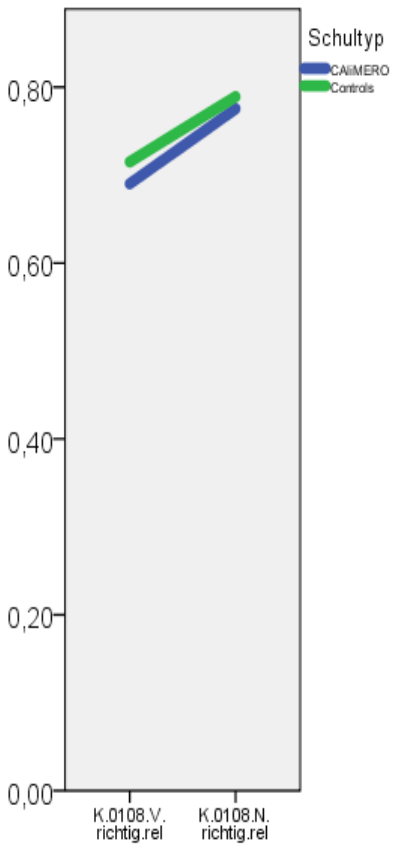

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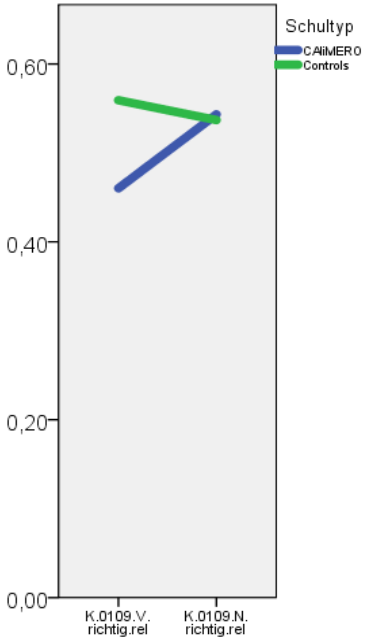

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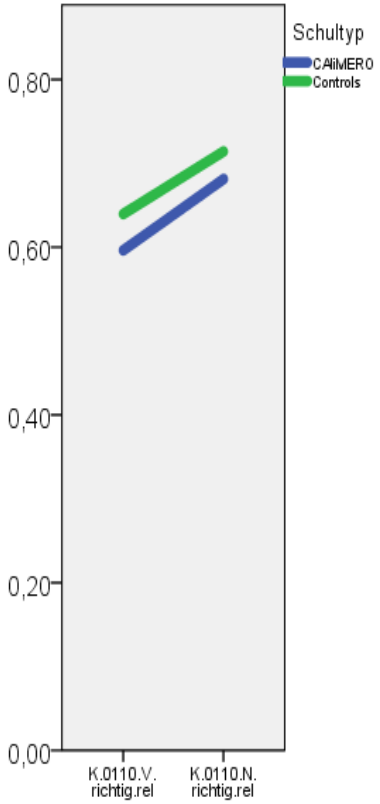

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Hattie(2009): 20-30% of the differences in performance can be explained by characteristics of the teacher.

On the basis of the data of the lesson journals we examine the links between the methodological structure of the lesson and mathematics achievement:

- How does frequent HC-use in class affect mathematics achievement?
- To what extent does regular mental maths reinforce the availability of basic mathematical knowledge?
- How does a certain diversity of teaching methods affect mathematics achievement?

Variety of method - a characteristic of good teaching

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One frequently quoted characteristic of qualitatively good teaching is a variety of method (Weinert 1997, Brophy 1999),

because, in the light of the

> *„existing variety of differences among the pupils in personality, learning style, competence, motivation, behaviour and performance differences a mono-teaching culture [would be] inappropriate or even unfair. But different learning targets also urgently call for different teaching methods "* (Helmke 2009, p. 260).

A diversity of methods does not guarantee per se effective teaching.

> *"There are two issues here: knowing when (Which teaching goals and curricular contents are suitable for which teaching methods?) and knowing for whom? (Which group of pupils benefits from a particular teaching method or suffers from it?)"* (Helmke 2009, p. 260).

An index for practical method diversity in the classroom

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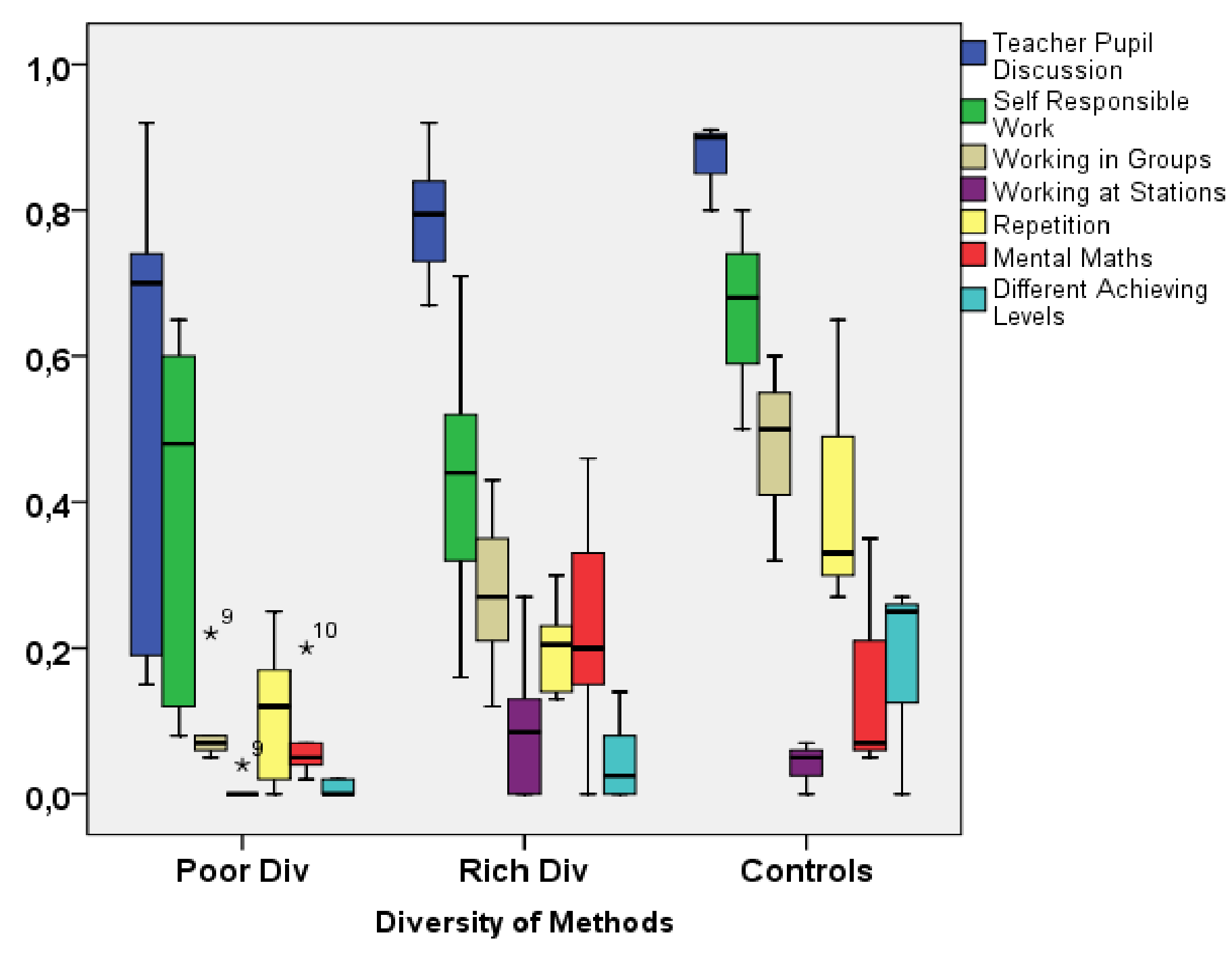
- Shannon-Index of biodiversity (Voleske et al. 2007)
- Modification: over-frequent use of a method and also the complete lack of method result in down-rating

  To calculate the value, the lesson journals provided details of frequency on the following aspects and methods of lesson design:

  - Teacher/pupil discussion
  - Self-responsible work
  - Working in groups
  - Working at stations
  - Repetition of covered materials
  - Mental maths (without calculator)
  - Different achieving levels for high-achieving and weak pupils

- We differentiate the project-classes in „method-rich“ (N= 10) and „method-poor“ (N=5) learning groups and compare with the controls(N=3)


1,0
0,8
0,6
0,4
0,2
0,0
9
10
9
Poor Div
Rich Div
Controls
Diversity of Methods
Teacher Pupil Discussion
Self Responsible Work
Working in Groups
Working at Stations
Repetition
Mental Maths
Different Achieving Levels

Method diversity and performance development

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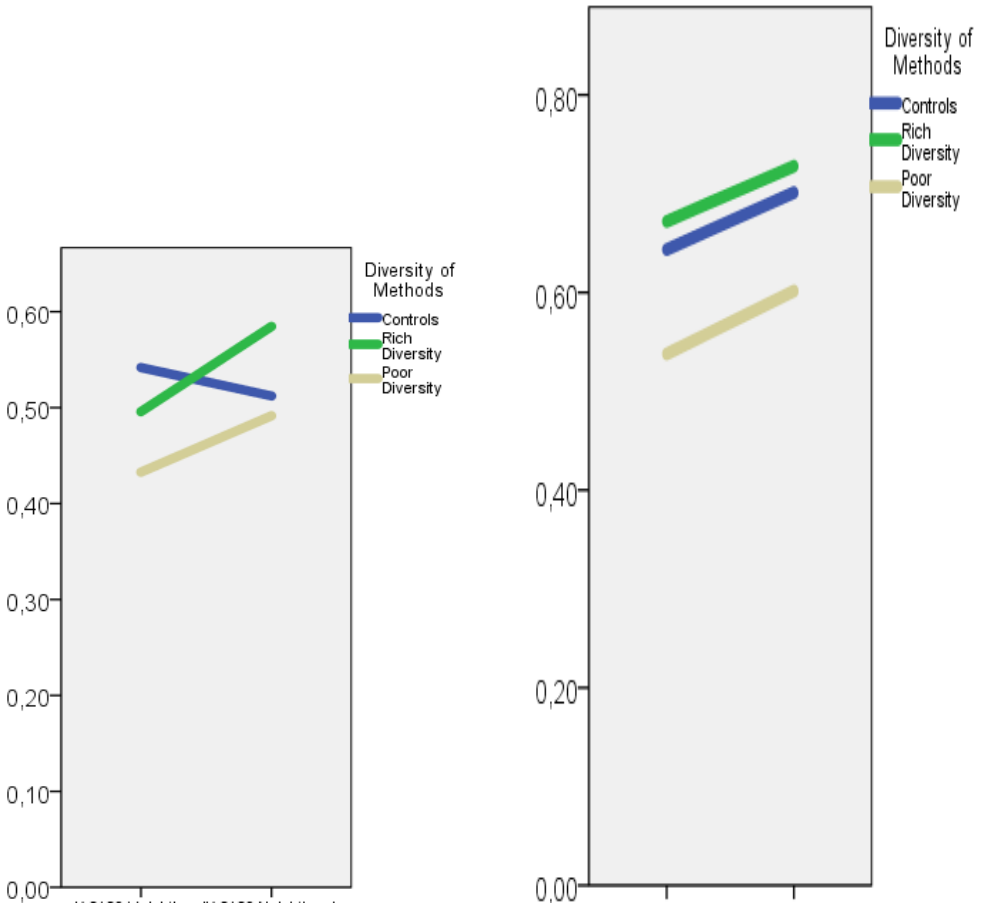
Basic-knowledge test
in year 9 and 10

rich diversity

poor diversity



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Frequency of mental maths and performance development

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N= 68 „normal frequence“ of mental maths

N= 81 low frequence

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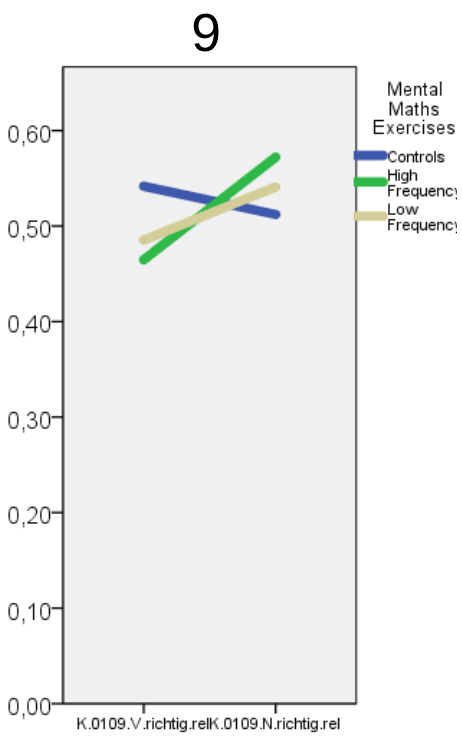

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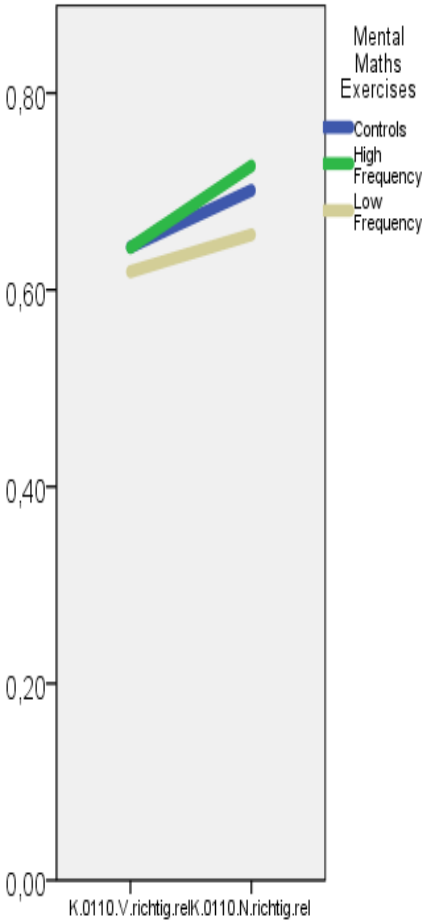

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Summary

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- CAliMERO refutes claims which are often propounded by leading educational institutions that deficits in basic mathematical knowledge can be assigned to HC use.
- No significant differences between project and control group with regard to performance development, but this does not mean that these effects do not exist.
- The performance increases of the boys and the girls are parallel.

- Diversity-rich learning groups reach a significant higher achievement level than diversity-poor groups. We guess, the achievement-level of the groups has influence on the methods choosen by the teachers. (Arbough et al. 2006)
- Teachers need their own experience with elements of a teaching concept with technology. Material based training the didactical-methodological competences of the teaching staff in this area is vital for effective technology-based teaching.

Key-idea of initial differentiation: Students can make a choice – not all students have to solve all given tasks.

Thank you!

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✓ ✓ ✓
✓ ✓ -
- ✓ ✓
✓ - ✓
✓ - -
- - ✓
- ✓ -
(-) – (-)

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Download presentation: www.math-learning.com

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