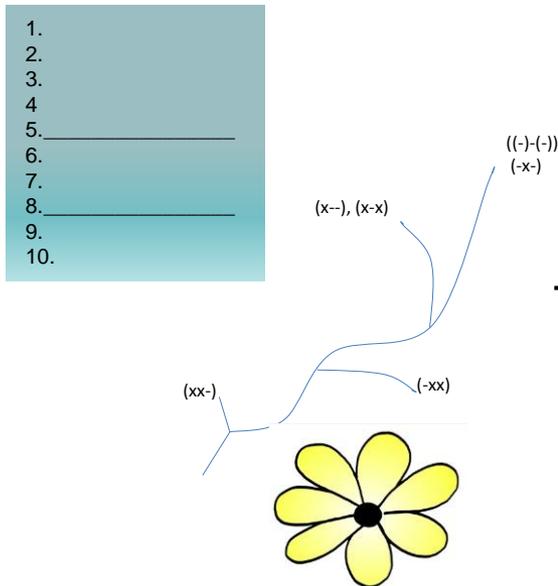


Eight target structure types of tasks as background for learning sourroundings



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www.math-learning.com

ICME 2012, TSG 31

✓ ✓ ✓
✓ ✓ -
- ✓ ✓
✓ - ✓
✓ - -
- - ✓
- ✓ -
(-) - (-)

The goal:



Presentation of a useful
(little, but very general and already old) theory of tasks
as a *target structure typification*

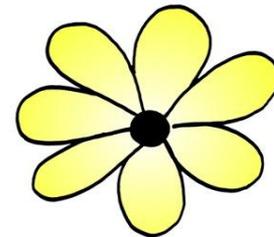
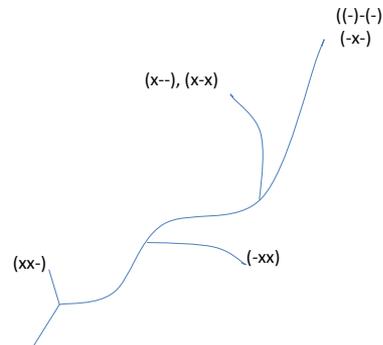
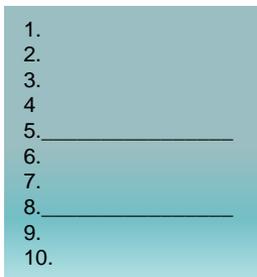
- for in-service teacher training
to change step by step
the „old task-culture“

$$\begin{aligned} 8. \text{ a) } & \frac{2x-3y}{2x+3y} - \frac{2x+3y}{2x-3y} + \frac{8x^2+18y^2}{4x^2-9y^2} \\ \text{ c) } & \frac{3p^2+1,5p-3}{18p^2-8} - \frac{2p+3}{15p+10} - \frac{p-1}{12p-8} \\ \text{ e) } & \frac{a^2+b^2}{2ab} - \frac{a}{a+b} - \frac{b}{a-b} + \frac{b^4-a^4+4a^3b}{2(a^3b-ab^3)} \\ \text{ g) } & \frac{x+y}{y} - \frac{x-y}{x} - \frac{4xy}{x^2+y^2} \\ \text{ i) } & \frac{3a-2b}{a^2-2ab+b^2} - \frac{2a+3b}{a^2-b^2} \\ \text{ k) } & \frac{7}{3k} - \frac{5}{k-3} + \frac{3}{k+1} + \frac{1}{k+5} \end{aligned}$$

$$\begin{aligned} \text{ b) } & \frac{5a-6b}{4a+4b} - \frac{2a-b}{3a-3b} - \frac{a^2-37ab+28b^2}{12a^2-12b^2} \\ \text{ d) } & \frac{2r}{rs-s^2} - \frac{2s}{r^2-rs} + \frac{r+s}{2rs} \\ \text{ f) } & \frac{1}{z-1} + \frac{1}{z+1} - \frac{2}{z^2-1} - 1 \\ \text{ h) } & \frac{1}{p^2-p} - \frac{p^2}{p+1} + \frac{1}{p} - \frac{2}{p^2-1} + p - 2 \\ \text{ j) } & \frac{m-3}{m+4} - \frac{m^2-9m-3}{m^2+m-12} + \frac{m-5}{m-3} \\ \text{ l) } & \frac{a}{a-b} - \frac{b^2}{a^2+ab+b^2} - \frac{a^2b}{a^3-b^3} \end{aligned}$$

Structure

- Theoretical background and the meaning of task
- Which different structure types of tasks exist and how they enhance sustained learning?
- Learning environments based on the structure types of tasks: „set of tasks“ and „blossom tasks“



The meaning of „Aufgabe“ (task)



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History:

Lenne'(1969) deals critically with the *task didactics* – a limited notion of task

$$\begin{array}{ll} 8. a) \frac{2x-3y}{2x+3y} - \frac{2x+3y}{2x-3y} + \frac{8x^2+18y^2}{4x^2-9y^2} & b) \frac{5a-6b}{4a+4b} - \frac{2a-b}{3a-3b} - \frac{a^2-37ab+28b^2}{12a^2-12b^2} \\ c) \frac{3p^2+1,5p-3}{18p^2-8} - \frac{2p+3}{15p+10} - \frac{p-1}{12p-8} & d) \frac{2r}{rs-s^2} - \frac{2s}{r^2-rs} + \frac{r+s}{2rs} \\ e) \frac{a^2+b^2}{2ab} - \frac{a}{a+b} - \frac{b}{a-b} + \frac{b^4-a^4+4a^3b}{2(a^3b-ab^3)} & f) \frac{1}{z-1} + \frac{1}{z+1} - \frac{2}{z^2-1} - 1 \\ g) \frac{x+y}{y} - \frac{x-y}{x} - \frac{4xy}{x^2+y^2} & h) \frac{1}{p^2-p} - \frac{p^2}{p+1} + \frac{1}{p} - \frac{2}{p^2-1} + p - 2 \\ i) \frac{3a-2b}{a^2-2ab+b^2} - \frac{2a+3b}{a^2-b^2} & j) \frac{m-3}{m+4} - \frac{m^2-9m-3}{m^2+m-12} + \frac{m-5}{m-3} \\ k) \frac{7}{3k} - \frac{5}{k-3} + \frac{3}{k+1} + \frac{1}{k+5} & l) \frac{a}{a-b} - \frac{b^2}{a^2+ab+b^2} - \frac{a^2b}{a^3-b^3} \end{array}$$

The meaning of „Aufgabe“ (task)

History:

Lenne'(1969) deals critically with the *task didactics* – a limited notion of task

Dietz (1981) developed the *leading task concept* - according to Davydov (1977) included a hierarchically structured task system with far-reaching scope and graduated subtasks

In the nineties the notion of "*open-ended maths tasks*" led to an enlargement of the task properties.

Now:

The goals of modern math lessons are

- a higher individualisation of students' learning and
- the sustained increase of competencies.

How and which tasks can help?



Tasks can take on *functions* on several levels:

(If the teachers work with the task in an adequate way and use the potential)

- tasks to develop or control *intelligent knowledge* (terms, theorems, approaches, strategies, application fields),
- tasks to obtain, enhance or identify *competencies to act* (recognition, description, connection, application, explanation of mathematical terms, correlations or proceedings),
- tasks to support the development and description of *meta competencies* (among others development of an adequate mathematical view, enhancement of self-assessment).

„Question“ or „exercise“ are used as synonyms for task.

But: „Learning task“ has long been established in the theory of activity as an individual target orientation.

Which different structure types of tasks exist and how they enhance sustained learning?



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The diversity of possible and required student activities in maths lessons makes an enlarged task notion seem advisable:

Following **Lompscher** et al (1985 – “*activity theory*”) we understand a task

- as an invitation to *develop learning activities*

The notion of **task** has now been located above the notion of **problem**.
Problems are considered as individually difficult **tasks**.

Which different structure types of tasks exist and how they enhance sustained learning?

The diversity of possible and required student activities in maths lessons makes an enlarged task notion seem advisable:

Following **Lompscher** et al (1985 – “*activity theory*”) we understand a task

- as an invitation to *develop learning activities*
with the components



- these three components are known (✓) or unknown (-)

✓ ✓ ✓
✓ ✓ -
- ✓ ✓
✓ - ✓
✓ - -
- - ✓
- ✓ -
(-) - (-)

Every task can be described by one of eight possible triples for the target structure.



✓

✓

✓

solved problem, example, pattern

✓

✓

-

easy task („basic task“)

-

✓

✓

inverse easy task (depends on the direction of learning the new mathematics)

Basic task:

-Simple mental arithmetic tasks: $12^2 = ?$

-Solve the quadratic equation $3x^2 + 7x = 8$ (necessary approach/algorithm is known)

Inverse easy task:

- Give a quadratic equation with the solutions 3 and -2. Can you find more than one?



Initial situation	Transformation	Final situation
-------------------	----------------	-----------------

✓

✓

✓

solved problem

✓

✓

-

easy task

-

✓

✓

inverse easy task

✓

-

✓

proving task, a strategy finding task

A proving task:

Why is the formula applicable for quadratic equations $f(x) = x^2 + px + q$?

Initial situation	Transformation	Final situation
-------------------	----------------	-----------------

✓	✓	✓	solved problem
---	---	---	-----------------------

✓	✓	-	easy task
---	---	---	------------------

-	✓	✓	inverse easy task
---	---	---	--------------------------

✓	-	✓	proving task, a strategy finding task
---	---	---	--

A proving task:

Why is the formula applicable for quadratic equations $f(x) = x^2 + px + q$?

Or:

A strategy finding task:

Frank is always winning the NIMM-game. **How does he proceed?**

The NIMM-game is played as follows: There are 20 matches on the table. Two players are playing against each other. The player who can take the last match – if **one, two or three matches** per move are taken – will be the winner.



Variation of the NIMM-game:



- How we can change the rules (or the winning strategy), if we have 25 matches?
- Find other questions or game-variations!

Student: When it is possible to win on the first position?

Initial situation	Transformation	Final situation
-------------------	----------------	-----------------

✓	✓	✓	solved problem
✓	✓	-	easy task
-	✓	✓	inverse easy task
✓	-	✓	proving task, a strategy finding task
✓	-	(-)	(difficult/open) determination task

Is the (tetrapack) milk carton for 1 l an optimal packaging?

Michael Todd Edwards` (open) example: (✓ - -)
With steps (in his article): (✓ - (-))





Initial situation	Transformation	Final situation	
✓	✓	✓	solved problem
✓	✓	-	easy task
-	✓	✓	inverse easy task
✓	-	✓	proving task, a strategy finding task
✓	-	(-)	(difficult/open) determination task
-	-	✓	(difficult/open) inverse determination task

Is the tetrapack milk carton for 1 l an optimal packaging?

A pond is planned with a surface of 10m².
What will be the dimensions of the pond?

What could be difficult?
- to make decisions (about the form of the pond...)



Initial situation	Transformation	Final situation
-------------------	----------------	-----------------

✓	✓	✓	solved problem
✓	✓	-	easy task
-	✓	✓	inverse easy task
✓	-	✓	proving task, a strategy finding task
✓	-	-	difficult task
-	-	✓	difficult inverse task
-	✓	-	a request to invent a task

Write a story including the three key questions of percentage calculation !



Initial situation	Transformation	Final situation	
✓	✓	✓	solved problem
✓	✓	-	easy task
-	✓	✓	inverse easy task
✓	-	✓	proving task, a strategy finding task
✓	-	-	difficult task
-	-	✓	difficult inverse task
-	✓	-	a request to invent a task
(-)	-	(-)	modelling task

How long does it take to change the water in the swimming pool?

Or:

Make a survey about....with your fellow students and present the results!

A useful theory...

On the basis of many years of experience in teacher training courses:
the orientation on these eight structure types leads to greater
methodical variety in the planning of teaching units,
and offers students the possibility to understand the learning
content far beyond formal reproduction and application.

A useful theory...



- On the basis of many years of experience in teacher training courses:
the orientation on these eight structure types leads to greater methodical variety in the planning of teaching units,
and offers students the possibility to understand the learning content far beyond formal reproduction and application.
- These 8 structure types may serve as a **guideline and construction heuristics** for the design of units for self-directed learning, learning by stages, expert puzzles or for the provision of learning conditions to stimulate cognitive activation.

A useful theory...



- On the basis of many years of experience in teacher training courses:
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and offers students the possibility to understand the learning content far beyond formal reproduction and application.
- These 8 structure types may serve as a guideline and construction heuristics for the design of units for self-directed learning, learning by stages, expert puzzles or for the provision of learning conditions to stimulate cognitive activation.
- The structure types of tasks are **useful to analyse**
 - learning environments in textbooks and find *gaps* in the diversity of tasks.
 - artefacts of teacher trainings (tests!)
 - the quality of E-Learning environments (training-software for basic skills)and help to explain special effects in students' reaction...

Example:

What is a good task for learning modelling?



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- Type (✓ --)

can produce different prioritisations:

Is the approach (or its variety) the scope of the task – or is the result of central interest – or both? Will the student become aware of this?

Example:

What is a good task for learning modelling?



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▪ Type (✓ --)

can produce different prioritisations:

Is the approach (or its variety) the scope of the task – or is the result of central interest – or both? Will the student become aware of this?

FERMI-tasks (beloved by educationalists for their creativity):

- *How long is a strip coming out of a tube of toothpaste?*
- *How many tennis balls can you fit in our classroom?*

Depending on the preferred cognitive style (Gregory, 2005):

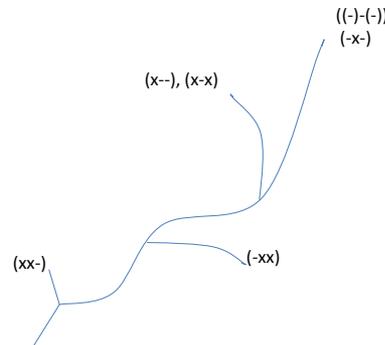
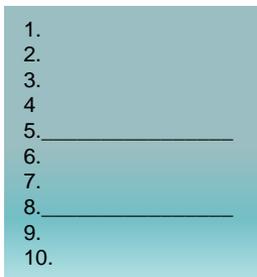
„Math have nothing to do with reality, who would fill a classroom with tennis balls?“

Quality criterion:

Choose modelling tasks with **relevant results** from the student's point of view and with interesting **different approaches** from a didactical point of view.

Structure

- The meaning of „Aufgabe“ (task)
- Which different structure types of tasks exist and how they enhance sustained learning?
- Learning environments based on the structure types of tasks: „set of tasks“ and „blossom tasks“





Background for our sample of task-formats is a concept of exercises (with empirical evidence) based on the activity theory with three phases:

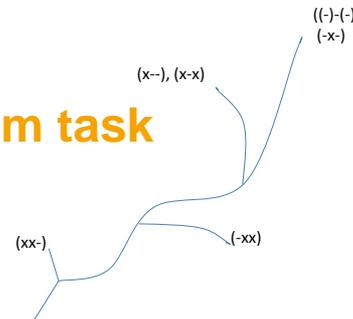
1. *First Exercises* for a basic understanding,

Task-set

- 1.
- 2.
- 3.
- 4
5. _____
- 6.
- 7.
8. _____
- 9.
- 10.

2. *Varied (intelligent) Exercises*

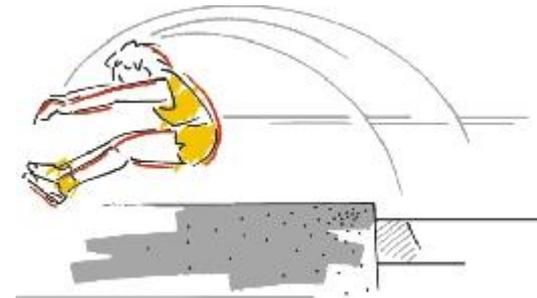
Blossom task



3. *Complex Exercises and Applications.*

Key-idea: Students can make a choice – not all students have to solve all given tasks

The goal:



Set of tasks for the *first exercises* to calculate a zero of linear functions



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Choose five tasks and you have 15min to complete them.

Find the point, where these linear functions cross the x-axis (zero): (✓ ✓ -)

1. $f(x) = x - 5$

2. $f(x) = 2x + 6$

3. $f(x) = -5x - 2.5$

(- ✓ ✓)

4. Draw a Graph of a linear function with the zero at $x = -5$

(✓ - -)

5. What are possible applications of a zero of a function? (What could the zero mean?)

Task-set for the *first exercises* to calculate a zero of linear functions



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Choose five tasks and you have 15min to complete them.

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1. $f(x) = x - 5$

2. $f(x) = 2x + 6$

3. $f(x) = -5x - 2.5$

(- ✓ ✓)

4. Draw a Graph of a linear function with the zero at $x = -3$

(✓ - -)

5. What are possible applications of a zero of a function? (What could the zero mean?)

6. Find the equation of two linear functions with a zero at $x = 4$

(- - ✓)

7. Find an equation of a linear function without any zeroes.

(- ✓ -)

8. Find a context which can be described with a linear function with a zero at $P(1,0)$.

Task-set for the *first exercises* to calculate a zero of linear functions



Choose **five tasks** and you have 15min to complete them (Grade 8).

Find the point, where these linear functions cross the x-axis (zero): (✓ ✓ -)

1. $f(x) = x - 5$

2. $f(x) = 2x + 6$

3. $f(x) = -5x - 2.5$

(- ✓ ✓)

4. Draw a Graph of a linear function with the zero at $x = -3$

(✓ - -)

5. What are possible applications of a zero of a function? (What could the zero mean?)

6. Find the equation of two linear functions with a zero at $x = 4$ (- - ✓)

7. Find an equation of a linear function without any zeroes.

(- ✓ -)

8. Find a context which can be described with a linear function with a zero at $P(1,0)$.

9. Why do linear functions not have more than one zero? (✓ - ✓)

10. Find an expression for the determination of the zero of $f(x) = mx + b$ and give the necessary condition for m , x and b . ((✓) - -)

Not a good example for a first exercise – why?



Nr.	Term	Quadrat. Ergänzung	Extremwert
1	$x^2 - 6x + 5 =$	$x^2 - 6x + 9 - 4 = (x - 3)^2 - 4$ (✓ ✓ ✓)	$T_{\min} = -4$ für $x = 3$
2	$x^2 + 4x + 4 =$		$T_{\min} =$ Für $x =$
3	$-x^2 - 2x + 1 =$		$T_{\min} =$ Für $x =$
4		$(x - 3)^2 - 4 =$	$T_{\min} =$ Für $x =$
5	$-0,25 x^2 + 4,5$		$T =$ Für $x =$
6		$-2(x \quad)^2 =$	$T_{\max} = 3$ Für $x = 1,5$

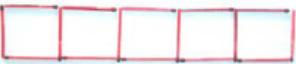
8. a) $\frac{2x-3y}{2x+3y} - \frac{2x+3y}{2x-3y} + \frac{8x^2+18y^2}{4x^2-9y^2}$	b) $\frac{5a-6b}{4a+4b} - \frac{2a-b}{3a-3b} - \frac{a^2-37ab+28b^2}{12a^2-12b^2}$
c) $\frac{3p^2+1,5p-3}{18p^2-8} - \frac{2p+3}{15p+10} - \frac{p-1}{12p-8}$	d) $\frac{2r}{rs-s^2} - \frac{2s}{r^2-rs} + \frac{r+s}{2rs}$
e) $\frac{a^2+b^2}{2ab} - \frac{a}{a+b} - \frac{b}{a-b} + \frac{b^4-a^4+4a^3b}{2(a^3b-ab^3)}$	f) $\frac{1}{z-1} + \frac{1}{z+1} - \frac{2}{z^2-1} - 1$
g) $\frac{x+y}{y} - \frac{x-y}{x} - \frac{4xy}{x^2+y^2}$	h) $\frac{1}{p^2-p} - \frac{p^2}{p+1} + \frac{1}{p} - \frac{2}{p^2-1} + p-2$
i) $\frac{3a-2b}{a^2-2ab+b^2} - \frac{2a+3b}{a^2-b^2}$	j) $\frac{m-3}{m+4} - \frac{m^2-9m-3}{m^2+m-12} + \frac{m-5}{m-3}$
k) $\frac{7}{3k} - \frac{5}{k-3} + \frac{3}{k+1} + \frac{1}{k+5}$	l) $\frac{a}{a-b} - \frac{b^2}{a^2+ab+b^2} - \frac{a^2b}{a^3-b^3}$

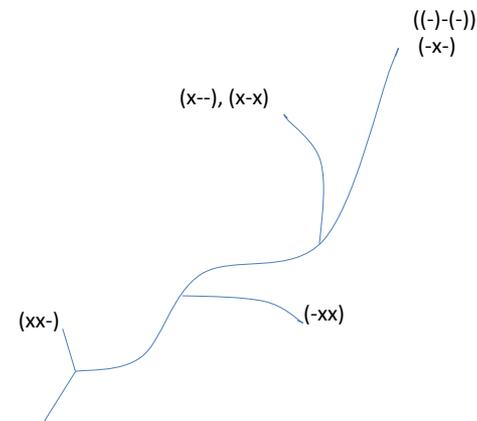
Blossom task as self-differentiating exercise form

Work 15 minutes on these tasks at choice

Square chains have been made with matches

a) Complete the table. (✓ ✓ -)

	Number of squares	Number of matches
	1	4
	2	7
	3	10
?	?	?
	5	16

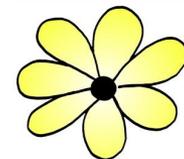


b) How many squares would 49 matches make? (- ✓ ✓)

c) Find an expression for the number of matches, if q is the number of the squares. (✓ - -)

d) Create another pattern with matches and find the expression for this situation. ((-) - (-))

Open question:



Better?

Blossom task as self-differentiating exercise form

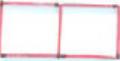
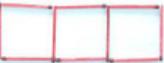


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Work 15 minutes on these tasks at choice:

Square chains have been made with matches

a) Complete the table.

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	1	4
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?	?	?
	5	16

b) How many squares would 49 matches make?

c) Find an expression for the number of matches, if q is the number of the squares.

d) Create another pattern with matches and find the expression for this situation.

Benefit:

Optional choice strengthens the self-responsibility of the students for their learning and gives a feeling of success

The same **context related** to all subtasks simplifies the discussion of the tasks

Low-threshold access makes the task accessible also for low attainers

Different task types allow individual and successive development of learning contents

Open subtasks enhance problem solving competencies and the creativity of students and support different levels of processing

Integral teaching concept (used in the project MABIKOM 2008- 2012)



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Differentiated teaching starts

Maintaining of basic knowledge

**Mental arithmetics
and diagnostics**

**Mental arithmetics
and diagnostics**

**Mental arithmetics
and diagnostics**

Rich exercise concept

Set of tasks

(initial and progressive exercises with
choice of difficulty)

**Long-term
homework**

Tasks with
supported
self-regulation

Blossom tasks

(gradual requirements
in self-differentiating
tasks)

Diagnostics of (self)competency

Learning report

(reflections on a subject
to support comprehension)

Check list

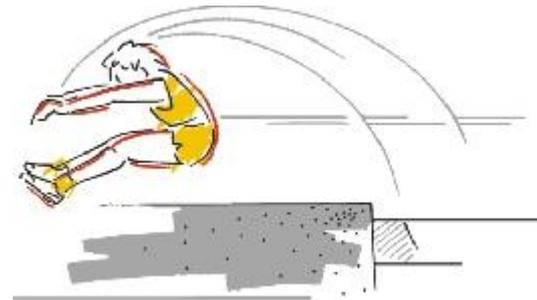
(self-assessment of
personal basic competency)

Learning control

Thank you!



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